Press

Columbia University
in the City of New York

GEOPHYSICAL LABORATORY
DEPARTMENT OF GEOLOGY

# Technical Reports on Seismology

## No. 3

## The Airy Phase of Shallow Focus Submarine Earthquakes

errata

p 24 $t > \Gamma/\beta_2$ instead of $t > /\beta_2$

Columbia University

New York, New York

# The Airy Phase of Shallow Focus Submarine Earthquakes

## Technical Report No. 3

by

Frank Press Frank Press
Maurice Ewing Maurice Ewing
Ivan Tolstoy Ivan Tolstoy

The research reported in this document has been made possible through support and sponsorship extended by the Geophysical Research Directorate of the Cambridge Field Station, AMC, U.S. Air Force, under Contract W-28-099 ac-396. It is published for technical information only and does not represent recommendations or conclusions of the sponsoring agency.

March 1949

Digitized by the Internet Archive
in 2020 with funding from
Columbia University Libraries

https://archive.org/details/airyphaseofshall00pres

## ABSTRACT

An extension of the theoretical work of Lamb and Pekeris shows that a prominent phase (Airy Phase) consisting of a group of waves of period 9-11 seconds, travelling across the ocean with a velocity of approximately 0.7 of the speed of sound in water should be present on the seismograms of shallow focus submarine earthquakes. This arrival corresponds to normal mode propagation at a stationary value of group velocity through the acoustic system consisting of the ocean and the rigid ocean bottom.

A phase fitting this description has been observed on the Milne-Shaw instruments at Bermuda for a series of Dominican Republic shocks. The Wenner seismographs at Huancayo recorded the Airy phase of a shock southwest of the Galapagos Islands.

The study of the Airy phase of a sufficient number of sea quakes would provide information concerning the nature of the ocean bottom since the latter affects both the periods and velocities of the Airy phase.

## Introduction

A number of seismograms of earthquakes occuring at sea were examined and it was found that for a series of these a new phase could be identified consisting of a group of waves of period 8-9 seconds, travelling across the ocean with a velocity of 0.7-0.8 of the speed of sound in water. It is believed that this arrival corresponds to normal mode propagation at a stationary value of group velocity through the acoustic system consisting of the ocean and the rigid ocean bottom.

Pekeris[1] predicted and later identified this phase on seismograms of explosive sounds in shallow water obtained by Worzel and Ewing.[2] In his theory of normal mode propagation in two liquid layers, Pekeris named this arrival the Airy phase and showed that its period depended on the depth of the water and the velocity contrast between the water and the bottom. That periods associated with a stationary value of group velocity would predominate at long distances had been recognized earlier by Jeffreys[3] and Stoneley.[4]

A theory for the origin of microseisms in which microseism groups are considered to be Airy phases was advanced by Press and Ewing.[5] This theory explains the mechanism of transmission

---

1. C. L. Pekeris, Theory of Propagation of Explosive Sounds in Shallow Water, in Propagation of Sound in the Ocean, Mem. No. 27, Geol. Soc. Amer., 1948
2. J. L. Worzel and Maurice Ewing, Explosive Sounds in Shallow Water, in Propagation of Sound in the Ocean, Mem. No. 27, Geol. Soc. Amer., 1948
3. Harold Jeffreys, On Surface Waves of Earthquakes, Mon. Not. Roy. Astron. Soc., Geophys. Suppl., Vol 1, pp 282-293, 1926
4. Robert Stoneley, Surface Waves Associated with 20° Discontinuity, Mon. Nat. Roy. Astron. Soc., Geophys. Suppl, Vol 4, pp39-43, 1937
5. Frank Press and Maurice Ewing, A theory of Microseisms with Geological Applications, Trans. Amer. Geoph. Union, Vol 29, No. 2, pp 163-174, 1948

of seismic energy from a storm at sea and correctly predicts that periods of three to nine sec will predominate, the period being determined by the depth of water and the elastic constants of the bottom.

Whether the source of seismic energy is a disturbance at the ocean surface produced by a hurricane, an explosive charge detonated within the ocean, or an earthquake beneath the bottom, the normal mode theory indicates that a prominent Airy phase should be observed at a distant point. Figure 1 , the seismogram from a shot of 5 lbs. in water 57 feet deep, at a distance of 4.5 nautical miles, illustrates the predominance of the Airy phase for shallow water explosions.

Although the treatment of the bottom as a liquid was adequate for the shallow water data to which Pekeris applied it, when one deals with submarine earthquakes in the deep ocean it becomes necessary to consider the bottom as an elastic solid. In this paper, the Pekeris theory is extended to include the case of an impulsive point source of compressional waves within the bottom which is taken as a solid.

## Propagation of Elastic Waves in the Ocean for an Impulsive Point Source beneath the Bottom

The analagous problem of the generation of Rayleigh waves at the free surface of a semi-infinite solid for a source within the solid was studied by Lamb.[6] Subsequent investigations by Stoneley[7] and Sezawa[8] dealt with the effect of the ocean on the transmission of seismic waves across the ocean bottom. Stoneley considered plane waves and calculated several values of phase and group velocity for long periods. Sezawa obtained an approximate solution for the propagation of cylindrical waves having great length compared to the depth of the water. Lee[9] calculated the amplitudes of Rayleigh waves for plane waves transmitted through a homogeneous medium covered by a superficial layer, assuming various values for the elastic constants of the two media. Scholte[10] gave a theory for the combined effects of gravity and compressibility in a layer of water in contact with an elastic solid bottom. Although Scholte's emphasis was not on propagation, he did prepare an excellent tabulation of phase velocities for the first made. Press and Ewing[11] presented complete curves of phase and group velocity for the first and second modes of motion in a liquid layer superposed on a semi-infinite solid bottom and noted the importance of stationary values of group velocity for elastic waves transmitted across the ocean.

---

[6] Horace Lamb, On the Propagation of Tremors over the Surface of an Elastic Solid, Phil. Trans., Roy. Soc. Lon., Series A, Vol. 203, pp 1-42, 1904

In the present paper we make particular use of the methods and results of Lamb and Pekeris to derive the normal mode solutions from which the wave motion at large distances from the source can be obtained.

---

[7] Robert Stoneley, The Effect of the Ocean on Rayleigh Waves, Mon. Not. Roy. Astron. Soc., Geophys. Suppl., Vol. 1, pp 349-356, 1926

[8] Katsutada Sezawa, On the Transmission of Seismic Waves on the Bottom Surface of an Ocean, Bull. Earth. Inst. Tokyo, Vol. 9, Pt. 2, pp 115-143, 1931

[9] A. W. Lee, Further Investigations of the Effect of Geological Structure upon Microseismic Disturbance, Mon. Not. Roy. Astron. Soc., Geophys. Suppl., Vol. 3, pp 238-252, 1934

[10] J. G. Scholte, Over Het Verband Tussen Zeegolven en Microseismen, I and II, Verslag Med. Akad. Wet., Vol. 52, pp 669-683, 1943

[11] Frank Press and Maurice Ewing, loc. cit.

Consider the propagation of elastic waves through the system consisting of a liquid layer of thickness H, density $\rho_1$ and compressional wave velocity $v_1$, superposed upon an infinitely thick, solid bottom of density $\rho_2$ whose compressional distortional wave velocities are respectively $\alpha_2$ and $\beta_2$. $\lambda_1$ is the compressibility of the liquid and $\lambda_2$, $\mu_2$ are Lamé's constants for the bottom, and are related to the elastic wave velocities as follows:

$$\left.\begin{aligned} V_1^2 &= \lambda_1/\rho_1 \\ \alpha_2^2 &= (\lambda_2 + 2\mu_2)/\rho_2 \\ \beta_2^2 &= \mu_2/\rho_2 \end{aligned}\right\} \text{-----------------} \quad (1)$$

The Cartesian coordinate system is chosen with the x, y axes in the free surface and the z axis vertically downward. Assuming symmetry about the z axis we have $r = \sqrt{x^2 + y^2}$ and denote the displacements perpendicular and parallel to that axis by q and w respectively. The subscripts 1 and 2 refer to the liquid layer and bottom respectively.

## Simple Harmonic Compressional Point Source within Bottom

We wish to determine the component displacements q and w due to a point source of compressional waves at $r = 0$, $z = d + H$. In particular we will seek the normal mode solutions which predominate at large distances from the source.

Following the procedure of Lamb, we assume simple harmonic motion $e^{i\omega t}$ and introduce the functions $\varphi(r,z)$ and $\psi(r,z)$ defined by the equations:

$$\left.\begin{aligned} q_1 &= \partial\varphi_1/\partial r \\ w_1 &= \partial\varphi_1/\partial z \\ q_2 &= \partial\varphi_2/\partial r + \partial^2\psi_2/\partial r\,\partial z \\ w_2 &= \partial\varphi_2/\partial z + \partial^2\psi_2/\partial z^2 + \omega^2/\beta_2^2\ \psi_2 \end{aligned}\right\} \tag{2}$$

The vertical stress $p_{zz}$ and the tangential stress $p_{zr}$ can be expressed in terms of q, w, $\varphi$, $\psi$ and the elastic constants follow:

$$P_{zz} = \lambda\nabla^2\varphi + 2\mu\,\partial w/\partial z \tag{3}$$

$$P_{zr} = \mu(\partial q/\partial z + \partial w/\partial r) \tag{4}$$

It is required that the functions $\varphi$ and $\psi$ satisfy the equations:

$$(\nabla^2 + \omega^2/V_1^2)\,\varphi_1 = 0 \tag{5}$$

$$(\nabla^2 + \omega^2/\alpha_2^2)\,\varphi_2 = 0 \tag{6}$$

$$(\nabla^2 + \omega^2/\beta_2^2)\,\psi_2 = 0\ , \tag{7}$$

where $\nabla^2 = \partial^2/\partial r^2 + \frac{1}{r}\partial/\partial r + \partial^2/\partial z^2$

It is convenient at this point to give the solution where a periodic vertical force with magnitude $ZJ_0(kr)$ per unit area acts along the interface between the two media. The functions $\varphi$ and $\psi$ must satisfy the boundary conditions:

$$(P_{zz})_1 = 0 \quad at \ z = 0 \tag{8}$$

$$(P_{zz})_2 - (P_{zz})_1 = Z J_0(\kappa r) \quad at \ z = H \tag{9}$$

$$(P_{zr})_2 = 0 \quad at \ z = H \tag{10}$$

$$w_1 = w_2 \quad at \ z = H \tag{11}$$

Typical solutions of (5), (6) and (7) are of the form:

$$\varphi_1 = A \sin(\xi z) J_0(\kappa r) \tag{12}$$

$$\varphi_2 = B e^{-\eta z} J_0(\kappa r) \tag{13}$$

$$\psi_2 = C e^{-\zeta z} J_0(\kappa r), \tag{14}$$

where the separation constants $\xi$, $\eta$, $\zeta$, obtained by substituting (12)-(14) in (5)-(7), are

$$\left.\begin{aligned} \xi^2 &= \omega^2/V_1^2 - \kappa^2 \\ \eta^2 &= \kappa^2 - \omega^2/\alpha_2^2 \\ \zeta^2 &= \kappa^2 - \omega^2/\beta_2^2 \end{aligned}\right\} \tag{15}$$

$\xi$, $\eta$, $\zeta$ being positive real or positive imaginary. The time factor $e^{i\omega t}$ is temporarily omitted. The boundary condition expressed by (8) has been satisfied by the choice of $\varphi_1$. If equations (12)-(14) are substituted in the boundary conditions given by (9)-(11) using (3)-(4), three equations for the constants A, B, C are obtained. Solving for A, B, and C we have after some reduction:

$$\left.\begin{aligned} A &= \frac{Z}{\mu_2}\,\frac{\omega^2}{\beta_2^2}\,\frac{\eta}{\xi\,F(K)\cos(\xi H)} \\ B &= \frac{Z}{\mu_2}\,\frac{(2K^2-\omega^2/\beta_2^2)\,e^{\eta H}}{F(k)} \\ C &= \frac{Z}{\mu_2}\,\frac{2\eta\,e^{\zeta H}}{F(K)} \end{aligned}\right\} \text{-------------------- } (16)$$

where F(K) is given by:

$$F(K) = \frac{\rho_1\omega^4\eta}{\rho_2\beta_2^4\xi}\tan(\xi H) - \left[4K^2\eta\zeta - (2K^2-\omega^2/\beta_2^2)^2\right] \text{ ----- } (17)$$

For a source of compressional waves beneath the interface at the point r = 0, z = H + d, the simplest solution which gives a compressional wave travelling radially outward in an unlimited solid is

$$\varphi_2 = \frac{e^{-i\frac{\omega}{\alpha_2}R}}{R}\,, \qquad \psi_2 = 0 \text{ ------------------- } (18)$$

where $R = \sqrt{x^2+y^2+(z-H-d)^2}$ denotes the radial distance from the point source. If an image is placed at r = 0, z = H - d, we obtain a solution for which the tangential stress at z = H vanishes, namely

$$\left.\begin{aligned} \varphi_2 &= \frac{e^{-i\frac{\omega}{\alpha_2}R}}{R} + \frac{e^{-i\frac{\omega}{\alpha_2}R'}}{R'} \\ \psi_2 &= 0 \end{aligned}\right\} \text{-----------------} (19)$$

where $R' = \sqrt{x^2+y^2+(z-H+d)^2}$. From the integral form of the Bessel function Lamb[12] showed that the above expression for $\varphi_2$ is equivalent to

$$\varphi_2 = \int_0^\infty \frac{e^{-\eta(z-H+d)}}{\eta} J_0(kr)k\,dk + \int_0^\infty \frac{e^{\eta(z-H-d)}}{\eta} J_0(kr)k\,dk$$

$$= 2\int_0^\infty \frac{\cosh[\eta(z-H)]}{\eta} e^{-\eta d} J_0(kr)k\,dk \;. \text{-----------------} (20)$$

From (3) and (4) we can write for z = H

$$\left.\begin{aligned} p_{zr} &= 0 \\ p_{zz} &= 2\mu_2\int_0^\infty \frac{(2k^2-\omega^2/\beta_2^2)}{\eta} e^{-\eta d} J_0(kr)k\,dk \end{aligned}\right\}. \text{-----------------} (21)$$

Referring to (9) we note that the condition for continuity of normal stress at the interface will be satisfied by superposing on (20) the solution obtained from (16) and (12)-(14) by putting

$$Z = -\frac{2\mu_2(2k^2-\omega^2/\beta_2^2)}{\eta} e^{-\eta d} k\,dk \text{ --------------} (22)$$

---

[12] Horace Lamb, loc. cit.

and integrating with respect to k between the limits 0 and $\infty$. It can be verified from (2), (12)-(14) and (16) that the condition for continuity of w across the interface will be satisfied. Thus we may write for the formal, steady state solution for a periodic compressional point source:

$$\varphi_1 = -2\frac{\omega^2}{\beta_2^2}\int_0^\infty \frac{(2k^2-\omega^2/\beta_2^2)\sin(\xi z)}{\xi F(k)\cos(\xi H)} e^{-\eta d} J_0(kr)\, k\, dk \text{ ----------(23)}$$

$$\varphi_2 = 2\int_0^\infty \frac{\cosh[\eta(z-H)]}{\eta} e^{-\eta d} J_0(kr)\, k\, dk$$

$$-2\int_0^\infty \frac{(2k^2-\omega^2/\beta_2^2)^2}{\eta F(k)} e^{-\eta d} e^{-\eta(z-H)} J_0(kr)\, k\, dk \text{ -----------(24)}$$

$$\psi_2 = -4\int_0^\infty \frac{(2k^2-\omega^2/\beta_2^2)}{F(k)} e^{-\eta d} e^{-\gamma(z-H)} J_0(kr)\, k\, dk \text{ -----------(25)}$$

From (2), (24) and (25) we may write for the displacements $q_H$, $w_H$ at the surface of the solid bottom (z = H):

$$q_H = -2\frac{\omega^2}{\beta_2^2}\int_0^\infty \frac{k}{F(k)}\left[\frac{\rho_1\omega^2}{\rho_2\beta_2^2}\,\frac{\tan(\xi H)}{\xi} - 2\gamma\right] e^{-\eta d} J_1(kr)\,k\,dk \text{ - - - - - - - - - - - (26)}$$

$$w_H = -2\frac{\omega^2}{\beta_2^2}\int_0^\infty \frac{(2k^2-\omega^2/\beta_2^2)}{F(k)}\, e^{-\eta d} J_0(kr)\,k\,dk \text{ - - - - - - - - - - - - - - - - - - - - - (27)}$$

Integrals of this general type have been evaluated by Lamb[13], Nakano[14], Sezawa[15] and Pekeris[16]. The procedure is to transform the path of integration to the complex k-plane. The solutions can then be expressed as the sum of the residues of the integrands and two integrals along branch lines corresponding to the branch points $k = \omega/\alpha_2$ and $k = \omega/\beta_2$. The residues, which diminish as $1/\sqrt{r}$ give the normal mode solutions, whereas the branch line integrals diminish as $1/r^2$, and become negligible for large r.

Following this procedure and neglecting for the while the contributions of the branch line integrals, we find for large values of r:

---

[13] Horace Lamb, loc. cit.

[14] H. Nakano, On Rayleigh Wave, Jap. Journ. Astron. and Geophys., Vol. II, No. 5, pp 233-326, 1925

[15] Katsutada Sezawa, Love-Waves Generated from a Source of a Certain Depth, Bull. Earth. Res. Inst. Tokyo, Vol. 13, Pt. 1, pp 1-17, 1935

[16] C. L. Pekeris, loc. cit.

$$q_H = 2i\pi \frac{\omega^2}{\beta_2^2} \sum_n \sqrt{\frac{2}{\pi k_n r}} \; \frac{k_n^2}{\frac{\partial F(k_n)}{\partial k}} \left[ \frac{\rho_1}{\rho_2} \frac{\omega^2}{\beta_2^2} \frac{\tan(\xi_n H)}{\xi_n} - 2\zeta_n \right] e^{-\eta_n d} e^{i(\omega t - k_n r - \pi/4)} \quad \text{----}(28)$$

$$w_H = 2\pi \frac{\omega^2}{\beta_2^2} \sum_n \sqrt{\frac{2}{\pi k_n r}} \; \frac{k_n}{\frac{\partial F(k_n)}{\partial k_n}} \left( 2k_n^2 - \omega^2/\beta_2^2 \right) e^{-\eta_n d} e^{i(\omega t - k_n r - \pi/4)} \quad \text{-----}(29)$$

The subscript n indicates that the quantity is to be evaluated at $k = k_n$, where $k_n$ are the roots of the period equation:

$$F(k) = 0 \quad \text{----------------------} \quad (30)$$

From the factor $e^{i(\omega t - k_n r - \pi/4)}$ it can be seen that the phase velocity $c_n$ can be expressed by

$$c_n = \omega / k_n \quad \text{----------------------------------}(31)$$

and (28), (29) can be written in the form

$$q_H = \frac{2\pi}{H^2} \sqrt{\frac{2}{\pi r}} \sum_n \frac{1}{\sqrt{k_n}} Q(k_n)\, e^{-\eta_n d} e^{i(\omega t - k_n r + \pi/4)} \quad \text{-------------} (32)$$

$$w_H = \frac{2\pi}{H^2} \sqrt{\frac{2}{\pi r}} \sum_n \frac{1}{\sqrt{k_n}} W(k_n)\, e^{-\eta_n d} e^{i(\omega t - k_n r - \pi/4)} \quad \text{-----------------} (33)$$

where

$$Q(k_n) = \frac{(k_n H)^2}{R(k_n)} \frac{c_n^2}{\beta_2^2} \left[ \frac{\rho_1}{\rho_2} \frac{c_n^2}{\beta_2^2} \frac{\sqrt{1 - c_n^2/\alpha_2^2}}{\sqrt{c_n^2/V_1^2 - 1}} \tan\left(k_n H \sqrt{c_n^2/V_1^2 - 1}\right) - 2\sqrt{1 - c_n^2/\alpha_2^2}\sqrt{1 - c_n^2/\beta_2^2} \right] \quad \text{---------------}(34)$$

$$W(k_n) = \frac{(k_n H)^2}{R(k_n)} \frac{c_n^2}{\beta_2^2} \left(2 - c_n^2/\beta_2^2\right) \sqrt{1 - c_n^2/\alpha_2^2} \qquad \text{- - - - - - - - - - - - - - - - - - - -(35)}$$

and

$$R(k_n) = \frac{\rho_1 c_n^4}{\rho_2 \beta_2^4} \left\{ \left[ 1 + \frac{(1 - c_n^2/\alpha_2^2)}{(c_n^2/V_1^2 - 1)} \right] \frac{\tan\left(k_n H \sqrt{c_n^2/V_1^2 - 1}\right)}{\sqrt{c_n^2/V_1^2 - 1}} - \frac{(k_n H)(1 - c_n^2/\alpha_2^2)}{(c_n^2/V_1^2 - 1)} \sec^2\left(k_n H \sqrt{c_n^2/V_1^2 - 1}\right) \right\}$$

$$- 4 \left\{ (3 - 2c_n^2/\alpha_2^2) \sqrt{1 - c_n^2/\beta_2^2} + \frac{(1 - c_n^2/\alpha_2^2)}{\sqrt{1 - c_n^2/\beta_2^2}} - 2(2 - c_n^2/\beta_2^2) \sqrt{1 - c_n^2/\alpha_2^2} \right\}$$

$$\text{- - - - - - - (36)}$$

Equation (30) takes the dimensionless form:

$$\tan\left(k_n H \sqrt{c_n^2/V_1^2 - 1}\right) = \frac{\rho_2 \beta_2^4}{\rho_1 c_n^4} \frac{\sqrt{c_n^2/V_1^2 - 1}}{\sqrt{1 - c_n^2/\alpha_2^2}} \left[ 4 \sqrt{1 - c_n^2/\alpha_2^2} \sqrt{1 - c_n^2/\beta_2^2} - \left(2 - c_n^2/\beta_2^2\right)^2 \right].$$

$$\text{- - - - - (37)}$$

This last equation defines an implicit relationship between the period $T_n = 2\pi/c_n k_n$ and the phase velocity $c_n$, with the elastic constants of the system as parameters. It is evident that the wave system given by (28) and (29) will be attenuated for $k_n$ complex, the degree of attenuation depending on the magnitude of the imaginary component. The energy loss associated with damped propagation is due to the refraction of waves from the liquid layer into the solid bottom. For the case $\alpha_2 > \beta_2 > V_1$ which is considered in this paper, undamped propagation will occur for the cases:

I. $\alpha_2 > \beta_2 \geq c \geq V_1$

II. $\alpha_2 > \beta_2 > V_1 \geq c$ .

Only case I will be considered in this paper.

Numerical calculation of the period from (37) shows it to be a multiple valued function of phase velocity, each value belonging to a distinct mode of propagation. The summations indicated in (28) and (29) are to be carried over all the modes $n = 1$ to $n = \infty$. The amplitude factors $Q(k_n)$ and $W(k_n)$ give the relative excitation of each mode as a function of the period. The factor $e^{-\eta_n d}$ shows the influence of the depth of the source on the amplitudes of the different modes.

It can be shown[17] that the period equation (37) expresses the condition of constructive interference between plane waves undergoing multiple reflection in the liquid layer at angles of incidence beyond the critical angle of reflection $\theta_c$, where $\sin\theta_c = V_1/\beta_2$. With this simplified point of view each mode represents a different order of interference and the disturbance at a distant point is obtained by the superposition of waves arriving at the point along the oblique ray paths (defined by the angle of incidence $\theta$) for which constructive interference occurs. For the case of plane waves the wave number k in the previous discussion can be expressed by $k = \frac{2\pi}{l}\sin\theta$, where l is the wavelength measured along an oblique ray path. The plane wave theory however fails to provide an adequate explanation of how waves from a source in the bottom can enter the liquid at angles of incidence beyond $\theta_c$.

---

[17] Frank Press and Maurice Ewing, loc. cit.

## Generalization for an Arbitrary Pulse

In an dispersive medium in which an arbitrary initial disturbance occurs, the energy associated with each period is known to propagate with the group velocity given by the well known formula

$$U = c + (kh)\frac{dc}{d(kh)} \quad \text{------------------} \quad (38)$$

The period equation was used to obtain the phase velocity as a function of kh. The group velocity was subsequently determined by numerical differentiation according to (38). The results of these computations for $c \geqslant v_1$ (case I above) are shown in Figures (2) and (3) where $c/v_1$ and $U/v_1$ are plotted as functions of the dimensionless parameter $\gamma = H/\ell = H/v_1T = kHc/2\pi v_1$ for the cases $\rho_2/\rho_1 = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 2v_1$ and $\rho_2/\rho_1 = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 3v_1$ which represent respectively the approximate conditions for a granitic and basaltic bottom. Since the functional dependence of kh on $c/v_1$ is not very sensitive to small changes in the value of $\rho_2/\rho_1$ we have taken a single value of this constant for both cases. For both cases two modes are plotted. In a previous paper[18] the first and second mode was considered to represent two distinct branches of the period equation, the second branch having higher modes. This distinction is artificial and unnecessary. The phase and group velocity of the first mode (n = 1) approach the velocity ($C_R$) of Rayleigh waves in the bottom layer as $\gamma \rightarrow 0$, or as the wavelength becomes very long in comparison to the thickness of the first layer. As $c/v_1$ and $U/v_1 \rightarrow 1$, $\gamma$ approaches a limiting value. For

---

[18] Frank Press and Maurice Ewing, loc. cit.

$c/v_1 < 1$ (case II above) the propagation reduces to that of a boundary wave (Stoneley wave) along the bottom interface. In the second mode ($n = 2$) the phase and group velocity equals the velocity of shear waves in the bottom ($c/v_1 = U/v_1 = 2$ or $c/v_1 = U/v_1 = 3$ as the case may be) for a limiting value of $\gamma$ corresponding to a cut-off period above which k is complex and damping occurs. For c and U close to $v_1$ the periods become infinitely small. Higher modes of propagation ($n = 3, 4 \ldots$) exist, each having the same cut-off velocity $c = U = \beta_2$, but increasingly shorter cut-off periods. In general the periods of the higher modes corresponding to a given phase or group velocity become progressively shorter. The group velocity of the first and second modes are further characterized by the existence of stationary values of group velocity, the importance of which will be discussed in a later paragraph.

Having obtained the solution for the steady state propagation of simple harmonic compressional waves originating in a point source it remains yet to obtain the solution for an arbitrary initial disturbance. If the time variation of the initial disturbance (19) at the source is f(t) represented by its Fourier transform

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} g(\omega)\, d\omega \text{ - - - - - - - - - - - - - - - - - - } (39)$$

the compound displacements $q_H'$ and $w_H'$ for an arbitrary pulse can be written as

$$q_H' = \frac{1}{H^2}\sqrt{\frac{2}{\pi r}} \sum_n \int_{-\infty}^{\infty} \frac{1}{\sqrt{K_n}} Q(K_n)\, g(\omega) e^{-\eta_n d} e^{i(\omega t - K_n r + \pi/4)} d\omega \text{ - - - - } (40)$$

$$w_H' = \frac{1}{H^2}\sqrt{\frac{2}{\pi r}} \sum_n \int_{-\infty}^{\infty} \frac{1}{\sqrt{K_n}} W(K_n)\, g(\omega)\, e^{-\eta_n d} e^{i(\omega t - K_n r - \pi/4)} d\omega \text{ - - - } (41)$$

where $k_n$ is a function of $\omega$ through (30) and $g(\omega)$ is given by

$$g(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\, dt \qquad (42)$$

Pekeris[19] was primarily interested in a source consisting of an exploding charge and he appropriately chose $f(t) = e^{-\sigma t}$ for $t > 0$ and $f(t) = 0$ for $t < 0$. Our primary interest is the impulse developed in an earthquake and for lack of more detailed information we choose $f(t)$ to vanish for all but infinitesimal values of $t$ in such a manner that $\int_{-\infty}^{\infty} f(t)\, dt = A$. With this choice equations (40) and (41) are given by:

$$q_H' = \frac{A}{H^2}\sqrt{\frac{2}{\pi r}} \sum_n \int_{-\infty}^{\infty} \frac{1}{\sqrt{k_n}}\, Q(k_n)\, e^{-\eta_n d} e^{i(\omega t - k_n r + \pi/4)}\, d\omega \qquad (43)$$

$$w_H' = \frac{A}{H^2}\sqrt{\frac{2}{\pi r}} \sum_n \int_{-\infty}^{\infty} \frac{1}{\sqrt{k_n}}\, W(k_n)\, e^{-\eta_n d} e^{i(\omega t - k_n r - \pi/4)}\, d\omega \qquad (44)$$

To evaluate these integrals we follow Pekeris in using Kelvin's approximate method of stationary phase. The method assumes that the exponential term oscillates rapidly over the range of integration, whereas the remainder of the intergrand is a slowly varying function of $\omega$. The principle contribution to the integral therefore occurs at the points for which $\partial f(\omega)/\partial\omega = 0$, where $f(\omega) = (\omega t - K_n r \pm \pi/4)$. An expansion of $f(\omega)$ about these points of stationary phase is made and formulae for the approximate evaluation of the integral are obtained. Jeffreys[20], Lamb[21], and Pekeris[22] derive and discuss the appropriate formulae. In evaluating the integrals (43) and (44) we shall make use of the equations and computations as given by Pekeris. The first of these can be written as follows:

---

[19] C. L. Pekeris, loc. cit.

$$\int_{-\infty}^{\infty} Y(\omega) e^{i(\omega t - K(\omega) r)} d\omega \simeq \sqrt{\frac{2\pi}{r\left|\frac{d^2 K(\omega_0)}{d\omega^2}\right|}}\, Y(\omega_0)\, e^{i(\omega_0 t - K_n(\omega_0) r \pm \pi/4)}$$

$$\simeq \sqrt{\frac{2\pi}{\left(\frac{H}{2\pi V_1}\right)\frac{r}{U^2}\left|\frac{dU}{d\gamma}\right|}}\, Y(\omega_0)\, e^{i(\omega_0 t - K(\omega_0) r \pm \pi/4)} \qquad ----(45)$$

where the upper or lower sign is to be taken in the exponential according as $dU/d\gamma$ is positive or negative and k, U, $dU/d\gamma$, $\omega_0$ are evaluated from the group and phase velocity curve for the values of r and t which satisfy $r/t = U$. In deriving the second equation of (45) use was made of the relations $\gamma = \frac{\omega H}{2\pi V_1}$, $U = \frac{d\omega}{dK}$, $\omega = cK$. The above formula is valid only when the following condition is met:

---

[20] Harold Jeffreys, loc. cit.

[21] Horace Lamb, Hydrodynamics, Cambridge University Press, 6th Edition, pp 395-398, 1932

[22] C. L. Pekeris, loc. cit.

$$\frac{1}{96\pi^2}\frac{H}{r}\left[-\frac{5\left(\frac{dZ}{d\gamma}\right)^2}{Z^3}+\frac{3\,\frac{d^2Z}{d\gamma^2}}{Z^2}\right] \ll 1 \qquad \text{---------} \quad (46)$$

where

$$Z = \frac{V_1^{\,2}}{H}\frac{d^2K}{d\omega^2} = -\frac{V_1}{2\pi U^2}\frac{dU}{d\gamma} \qquad \text{-------------} \quad (47)$$

This condition is violated for small r and for points near a stationary value of group velocity. In the latter case we use an approximate formula devised by Airy:

$$\int_{-\infty}^{\infty} Y(\omega)\,e^{i(\omega t - K(\omega)r)}\,d\omega \simeq \frac{2\pi}{3^{2/3}}\,\frac{E(v)\;Y(\omega_0)\;e^{i(\omega_0 t - K(\omega_0)r)}}{r^{1/3}\left[\frac{H^2}{2\pi V_1^{3}}\left|\frac{dZ}{d\gamma}\right|\right]^{1/3}} \qquad \text{----} \quad (48)$$

where

$$E(v) = v^{1/3}\left[J_{-1/3}(v) + J_{1/3}(v)\right] \qquad \begin{cases} t < \frac{r}{U_{min}} \\ t > \frac{r}{U_{max}} \end{cases} \qquad \text{------} \quad (49)$$

$$E(v) = v^{1/3}\left[I_{-1/3}(v) - I_{1/3}(v)\right] \qquad \begin{cases} t > \frac{r}{U_{min}} \\ t < \frac{r}{U_{max}} \end{cases} \qquad \text{---------} \quad (50)$$

and

$$V = \frac{4\sqrt{\pi}}{3\sqrt{\left|\frac{dZ}{d\gamma}\right|}}\left(\frac{r}{H}\right)\left|\gamma - \gamma_m\right|^{3/2} \quad \text{- - - - - - - - - - - - - -} (51)$$

$$\gamma = \frac{tV_1}{r} - 1 \quad , \quad \gamma_m = \frac{V_1}{U_0} - 1 \quad \text{- - - - - - - - - - - - - - -} (52)$$

Only the real part of the integral in (48) has been considered. In these equations the subscript o indicates that the quantity to which it is attached is to be evaluated at a stationary value(min or max) of group velocity. The use of (48) is justified only when

$$-\frac{1}{(2\pi)^{2/3}}\left(\frac{H}{r}\right)^{1/3}\frac{\left(\frac{d^2Z}{d\gamma^2}\right)G(v)}{\left|\frac{dZ}{d\gamma}\right|^{4/3}E(v)} << 1 \quad \text{- - - - - - - - - - - - - -} (53)$$

where

$$G(v) = \frac{3^{1/3}}{4}\left\{-\frac{2}{3}v^{2/3}\left[J_{-2/3}(v) - J_{2/3}(v)\right] + \frac{1}{2}v^{5/3}\left[J_{-1/3}(v) + J_{1/3}(v)\right]\right\} \quad \begin{matrix} t < \frac{r}{U_{min}} \\ t > \frac{r}{U_{max}} \end{matrix} \quad \text{- - - -} (54)$$

$$G(v) = \frac{3^{1/3}}{4}\left\{-\frac{2}{3}v^{2/3}\left[I_{-2/3}(v) - I_{2/3}(v)\right] + \frac{1}{2}v^{5/3}\left[I_{-1/3}(v) + I_{1/3}(v)\right]\right\} \quad \begin{matrix} t > U_{min} \\ t < U_{max} \end{matrix} \quad \text{- -} (55)$$

We now apply these results to equations (43) and (44). In order to compute the displacements $q'_H$ and $w'_H$, one first finds the contribution of the nth mode at a given time t and range r by determining $\omega$, , $c/U_1$, kH, $dU/V_1/d\gamma$ from the phase and group velocity curves appropriate to the medium such that r/t = U.

$q_H'$ and $w_H'$ are then obtained by summing over all modes:

$$q_H' = -\frac{4V_1A}{H^2r}\sum_n \frac{Q(k_n)\,e^{-\eta_n d}}{\sqrt{\frac{\gamma}{C_n/V_1}\,\frac{1}{U_n^2/V_1^2}\left|\frac{dU_n/V_1}{d\gamma}\right|}}\,\sin(\omega t-k_nr)\,, \quad \frac{dU_n/V_1}{d\gamma}>0 \quad ---- (56)$$

$$= \frac{4V_1A}{H^2r}\sum_n \frac{Q(k_n)\,e^{-\eta_n d}}{\sqrt{\frac{\gamma}{C_n/V_1}\,\frac{1}{U_n^2/V_1^2}\left|\frac{dU_n/V_1}{d\gamma}\right|}}\,\cos(\omega t-k_nr)\,, \quad \frac{dU_n/V_1}{d\gamma}<0 \quad --- (57)$$

$$w_H' = \frac{4V_1A}{H^2r}\sum_n \frac{W(k_n)\,e^{-\eta_n d}}{\sqrt{\frac{\gamma}{C_n/V_1}\,\frac{1}{U_n^2/V_1^2}\left|\frac{dU_n/V_1}{d\gamma}\right|}}\,\cos(\omega t-k_nr)\,, \quad \frac{dU_n/V_1}{d\gamma}>0 \quad --- (58)$$

$$= \frac{4V_1A}{H^2r}\sum_n \frac{W(k_n)\,e^{-\eta_n d}}{\sqrt{\frac{\gamma}{C_n/V_1}\,\frac{1}{U_n^2/V_1^2}\left|\frac{dU_n/V_1}{d\gamma}\right|}}\,\sin(\omega t-k_nr)\,, \quad \frac{dU_n/V_1}{d\gamma}<0 \quad -- (59)$$

provided r is large and t is sufficiently removed from the value $t_o = r/U_o$, where $U_o$ denotes a stationary value of group velocity. We have added to the expressions (43) and (44) their complex conjugates in deriving (56) - (59), since Q and W are even functions of $\omega$, $f(\omega)$ is an odd function of $\omega$, and there are two stationary points $\pm\omega$ where $df(\omega)/d\omega = 0$.

Near the time $t_0$ corresponding to a stationary value of group velocity (Airy phase) one first determines $\gamma$, $\omega$, k, c, Z, $dZ/d\gamma$ at this point from the phase and group velocity curves appropriate to the medium. As before, $q_H'$ and $w_H'$ are then computed by summing over all modes:

$$q_H' = \frac{4 V_1 A}{3^{2/3} H^2 r^{5/6}} \sum_n \frac{Q(k_n)\, e^{-\eta_n d}\, E(v)}{\sqrt{K/2\pi}\left[\frac{H^2}{2\pi}\left|\frac{dZ}{d\gamma}\right|\right]^{1/3}} \cos(\omega t - k_n r + \pi/4) \text{ - - - - } (60)$$

$$w_H' = \frac{4 V_1 A}{3^{2/3} H^2 r^{5/6}} \sum_n \frac{W(k_n)\, e^{-\eta_n d}\, E(v)}{\sqrt{K/2\pi}\left[\frac{H^2}{2\pi}\left|\frac{dZ}{d\gamma}\right|\right]^{1/3}} \cos(\omega t - k_n r - \pi/4) \text{ - - - - } (61)$$

provided r is large and t sufficiently close to $t_0$ for condition (53) to hold. The factors E(v) from which the envelope of the Airy phase can be determined and G(v)/E(v) from which the range of validity of (60), and (61) can be found was plotted by Pekeris and is reproduced here as Figure 4.

Equations (56) through (61) have been used to compute the amplitudes of the component displacements at the bottom interface for an impulsive point source of compressional waves located at a depth beneath the interface equal to the depths of the water (d = H) and at distances of 1000 km. and 10,000 km. respectively. The results for the first and second modes are presented in Figures 5 to 8 for the case

$$\frac{\rho_2}{\rho_1} = 2.5, \quad \alpha_2 = \sqrt{3}\beta_2, \quad \beta_2 = 3V_1, \quad H = 5 \text{ km}, \quad r = 1000 \text{ km}, \quad 10,000 \text{ km}.$$

In these figures the amplitudes are plotted as functions of the dimensionless parameter $\frac{t - t_\beta}{t_\beta}$ where t denotes the time after the initial impulse and $t_\beta$ is the travel time for shear waves in the bottom. The travel time of waves of any given period in the first two modes can be computed from Figure 9.

In order to show the effect of the depth of the source on the displacements $q_H'$ and $w_H'$ the amplitudes were computed for a source located in the bottom layer close to the interface (d = o). The resultant amplitudes in the first two modes for the two cases d = o and d = H are plotted against $\gamma = H/l$ in Figures 10 to 13.

## Discussion of the Normal Mode Solutions

In an earlier section the formal steady state solutions (26) and (27) were expressed as the sum of the residues of the integrands and two integrals along branch lines corresponding to the branch points $c = \alpha_2$ and $c = \beta_2$. The residues lead to the normal mode solutions which predominate at large distances because they diminish only as $1/\sqrt{r}$ . Now the normal mode solutions vanish at $c = U = \beta_2$ and do not exist for $c > \beta_2$ . In this region the branch line integrals contribute waves travelling with the speed of compressional and shear waves in the bottom, diminishing rapidly in amplitude with distance as $1/r^2$.

At a time $t > \beta_2$ after the initial impulse (i.e., immediately after the arrival of shear waves) the normal mode solutions predominate providing $r$ is large. Thus the ground motion in the first two modes due to a distant impulsive point source of compressional waves having a moderately broad spectrum and located within the bottom can be obtained with the aid of Figures 3 and 5-9.

In the first mode the group velocity approaches the velocity of Rayleigh waves ( $c_R = .9194\beta_2$ ) as $\gamma \rightarrow 0$ . The first arrival consists of long period Rayleigh waves whose amplitudes and period vary with time according to the graphs of Figures 5, 6 and 9. At the time $t = \frac{r}{v_1}$ a short period wave travelling with the speed of sound in water arrives*. According to Figures 10, 11 amplitudes associated with this wave are particularly small unless the source is located close to the interface. This follows from the factor $e^{-\nu_n d}$ in equations (56)-(61) which gives the dependence of the normal mode amplitudes on the depth of the source beneath the interface. Since

---

* The group velocity curves show an upper frequency limit for these waves. For higher frequencies the solutions reduce to boundary waves propagated along the interface in a manner analagous to Stoneley waves.

$$\eta_n d = \kappa_n H\left(\frac{d}{H}\right)\sqrt{1-c_n^2/\alpha_2^2} = \frac{2\pi}{c/v_1}\gamma_n\left(\frac{d}{H}\right)\sqrt{1-c_n^2/\alpha_2^2}$$

the amplitudes of the shorter period waves ( $\gamma_n$ large) are cut down if the source has any depth whatever. For source depths greater than several times the depth of water the amplitudes of all but the longest period waves are reduced to insignificantly small values.

For $t > r/v_1$ the short period and long period branches of the group velocity curve approach each other and finally merge at a minimum value. The Airy phase associated with this stationary value of group velocity consists of waves of large amplitude having a prescribed period and velocity. Since waves propagated at a stationary value of group velocity decrease with distance as $1/r^{5/6}$ (equations 60 and 61) and waves propagated at other values of group velocity diminish as 1/r, the relative amplitude of the Airy phase increases with distance. This effect is shown in Figures 5 and 6 where the amplitudes have been computed for the two ranges r = 1000 km. and r = 10,000 km.

The ground motion in the second mode (Figures 7, 8, 12, 13) begins with waves arriving with a cut-off period at a speed $U=\beta_2$. The amplitudes are zero at the onset and thereafter increase. At a time corresponding to propagation at the maximum value of group velocity* large amplitude waves arrive with an abrupt onset. Thereafter these waves separate into several simultaneous arrivals of different periods and amplitudes. At the time $t = r/v_1$ a short period wave ( $\gamma \rightarrow \infty$ ) arrives travelling with the speed of sound in water. As was the case in the first mode, the amplitude of this phase is negligibly small unless the source is located at or very near to the interface. For $t > r/v_1$ the two arrivals corresponding to two branches of the group velocity curve approach each other and merge at a minimum value of group velocity, producing an additional Airy phase in the second mode. Since the second mode Airy phase occurs at a shorter period, its amplitude is smaller than that of the first mode if the source depth is of the order of the thickness of the liquid layer or greater.

---

* Pekeris attaches the name Airy phase in reverse to maximum values of group velocity.

Table I gives the period T and group velocity $U/v_1$ associated with each of the stationary values of group velocity of the first two modes for the two cases a) $\rho_2/\rho_1 = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 2V_1$, $V_1 = 1520$ meters/sec, $H = 5000$ meters,

and b) $\rho_2/\rho_1 = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 3V_1$, $V_1 = 1520$ meters/sec, $H = 5000$ meters.

TABLE I

Periods and Group Velocities of Airy Phases

| | | $\gamma$ | T | $U/V_1$ |
|---|---|---|---|---|
| Case a | First Mode | .33 | 10. sec. | .80 |
| | Second Mode | .49 | 6.7 | 1.7 |
| | | .98 | 3.4 | .75 |
| Case b | First Mode | .28 | 12 | .64 |
| | Second Mode | .46 | 7.2 | 2.6 |
| | | .82 | 4.0 | .56 |

The minimum value of group velocity in the second mode indicated by the dashed lines in Figures 2 and 3 is not included in Table I since its exact position has not been calculated. We infer that amplitudes associated with this point are not significantly large because of its proximity to the cut-off period and its extremely sharp curvature.

The ratio of vertical to horizontal displacement for the first two modes is plotted as a function of $\gamma$ in Figure 14. It is seen that for $\gamma \to 0$ the ratio $\frac{w}{q}$ approaches the correct value for Rayleigh waves propagated along the surface of a semi-infinite elastic solid for which $\lambda = \mu$.

From equations (2), (3), and (23) it can be seen that the factor sin $(\xi_n z)$ gives the vertical variation of pressure and horizontal displacement and cos $(\xi_n z)$ gives the vertical variation of vertical displacement in the liquid layer. Nodes of pressure correspond to antinodes of vertical displacement and vice versa. In Figure 15, the vertical pressure variation in the liquid layer is presented as a function of $\gamma$ for the first two modes.

The ground motion at a point is obtained by the superposition of the contributions of all modes. Our discussion thus far has been limited to the first two modes. The periods of the higher modes corresponding to a given group velocity become progressively shorter. For source depths of the order of a fraction of the depth of water or more, the higher modes will contribute little to the ground motion because of the factor $e^{-\eta_n d}$. In addition, the contributions of the short period higher modes will be cut down because of their greater susceptibility to scattering by an irregular ocean bottom. The poor short period response characteristics of intermediate and long period seismographs will further discrimate against the higher modes.

The theory and computations described represent an earthquake by an impulsive point source of compressional waves. Actually an earthquake is neither of these. A true source is usually extended and rich in distortional waves.

Despite the highly idealized assumptions concerned the nature of an earthquake source, it is believed that the normal mode theory given above is sufficiently adequate to explain the salient features on the seismograms of shallow focus submarine earthquakes.

Thus far only the Airy phase of the first mode and the short period water wave phase ("T" phase) have been observed. In the next section the data on the Airy phase is presented. The "T" phase has been described in an earlier report.[23]

---

[23] The "T" Phase of Shallow Focus Submarine Earthquakes, Technical Report No. 1 by Ivan Tolstoy, Maurice Ewing and Frank Press.

## The Airy Phase Observed at Bermuda and Huancayo

### 1. Bermuda.

The Bermuda seismological station is situated at 32°23'N, 64°41'W. It is equipped with a Milne-Shaw, two components, oriented NE-SW and NW-SE. The free period is given as 12 sec and the magnification as 250 by the USCGS seismological bulletin.

A series of records was examined in the hope of detecting late arrivals of 9 - 12 sec period. Such arrivals were detected on all readable seismograms of the Dominican Republic shocks, but usually not for shocks occuring elsewhere in the Atlantic. Thus a series of Mid-Atlantic Ridge quakes were studied. including the large shocks of April 11, 1946 (1°S, 14 1/2°W, Pasadena magnitude 7.2) and November 25, 1941 (37 1/2°N, 18 1/2°W, magn. 8 1/4), and with one exception, all gave negative results. We will dwell upon this point later.

Table II gives the results obtained for a series of Dominican Republic shocks. H is the origin time, $M_A$ gives the arrival time of the maximum amplitude wave of the group interpreted as being the Airy phase. It will be noted that in some cases these times appear to differ somewhat for the two components. The column marked t.t gives the total travel time of the Airy phase in minutes and seconds. $A_A$ gives the maximum double trace amplitude of the Airy phase, $A_p$ and $A_s$ give those of the P and S groups (in cm). T gives the period of the group on the seismogram in seconds. This, as well as the arrival times are only approximate, since the paper velocity of these seismograms is only 8mm/min. However they are sufficient for the present purposes.

## TABLE II

| Date | H | $M_A$NW | $M_A$NE | t.t. NW | t.t. NE | T | $A_A$ NW | $A_A$ NE | $A_p$ NW | $A_p$ NE | $A_S$ NW | $A_S$ NE | Magn |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Aug 7 46 | 18:26:30 | 18:51:45 | 18:51:05 | 25:15 | 24:35 | 8-9 | 0.2 | 0.2 | 0.05 | 0.05 | 0.7 | 0.5 | |
| Aug 8 46 | 17:24:07 | 17:49:40 | 17:48:48 | 25:33 | 24:39 | 8-9 | 0.5 | 0.5 | 0.1 | 0.05 | 1.8 | 1.2 | |
| Aug 9 46 | 08:25:40 | 08:50:20 | 08:50:20<br>&<br>08:51:40 | 24:40 | 24:40<br>&<br>26:00 | 8-9 | 0.45 | 0.3<br>&<br>0.3 | 0.05 | 0.05 | 0.7 | 0.6 | |
| Aug 21 46 | 19:17:44 | 19:41:05 | 19:40:45 | 23:21 | 23:01 | 8-9 | 2.0 | 1.8 | 0.4 | 0.6 | 10.0? | 8.6 | 6 1/2- |
| Oct 4 46 | 14:45:32 | 15:10:03<br>&<br>15:09:05 | 15:10:03 | 24:31<br>&<br>23:33 | 24:31 | 8-9 | 1.2 | 1.5 | 0.4 | 0.6 | 2.8 | 2.6 | 7 |
| Jul 30 43 | 01:02:32 | 01:26:16 | 01:26:41 | 23:44 | 24:09 | 9-10 | 0.7 | 0.6 | 0.2 | 0.2 | 2.8 | 2.5 | |

This list includes only shocks of magnitude 7 or less. The large shocks, such as those of Aug 8, 1946, (magn. 7.6) and Aug 4, 1946, (magn. 8), July 29, 1943, (magn. 73) and others are not included. The reason for this is the impossibility of distinguishing the phase we are studying, due to the very large trace amplitudes and consequent confusion and fading of the traces.

## Arrival times and velocities.

Let us see now what these arrival times correspond to. The times given in Table II were read for the maximum amplitude of the group of waves interpreted as Airy phases. Such a procedure is justified by the theory of wave propagation in dispersive media: for a stationary value of group velocity there is a gradual build up of the amplitudes until the arrival of the corresponding wave length, at which point the amplitude is maximum.

For purposes of comparison of the arrival times of this phase with those of the classical earthquake phases, it would be necessary to pick the time at the beginning of the group. For this one should subtract roughly 2 min from the preceeding arrival times.

The first remark that we shall make is that, even with the latter provision, the travel times are much too large to be any of the known tabulated earthquake phases(24) such as either multiply reflected body waves, Love and Rayleigh waves. They are too large for ScS even if this could be observed at such a short distance from the epicenter.

---

(24) Harold Jeffreys and K. E. Bullen, Seismological Tables, British Association for the Advancement of Science, 1940

Also these phases are not aftershocks, as could easily be verified by examining the records obtained at seismological stations having instruments with different characteristics (such as the Weston and Fordham Benioffs). As the reader may verify, (25) (26) the published lists of aftershocks do not indicate any shocks whose arrival times would fit those of the Airy Phase.

We purposely did not include the aftershock of Aug 7, 1946, H = 19:21:32 in our list, since it was followed by another shock 15 min 35 sec later which did interfere with the Airy phase of the former.

We see therefore that this arrival is clearly a new phase, which cannot be explained by any of the standard types of elastic waves transmitted through the earth's outer or inner layers.

Figure 16 a,b reproduces seismograms for both components of the shocks of Aug 8, 1946, (H = 17:24:07) and Aug 9, 1946, (H = 08:25:40). The phase in question is seen to stand out very conspicuously from the late coda. On all records examined it stands out with the same perfect clarity, and even for shocks much smaller than any of those enumerated it is possible to detect it, though it is not easy to get good times.

To obtain the velocity of this phase we should have a fairly accurate idea of the distance from the epicenter. Unfortunately, it is not possible to obtain the accurate readings of the P-S interval necessary for this purpose from this type of seismogram. We will have to be content with an approximate evaluation.

---

(25) N.E.S.A. Bulletin, Aug. 1946

(26) Bull. Seism. Lab., Cal. Inst. of Tech., July-Sept, 1946

The largest of the shocks listed in Table II is that of Oct 4, 1946, H = 14:45:32. Its epicenter is given by Gutenberg as 18°45'N 68°30'W. This means $\Delta = 14°03'$. Taking the travel time as 24min 31sec, we get a velocity of 3480 feet/sec which is 0.7 of the speed of sound in water.

Considering the epicenter to have been determined to within ± 1°, which is a reasonable estimate, the above velocity is accurate to within ± 250 feet/sec, i.e., it is somewhere between 0.75 and 0.65 of the velocity of sound in water.

The period of this phase being on the record of the order of 9 sec, it is seen that it is in approximate agreement with the theoretically predicted Airy phase. It might be necessary to assume a slightly higher bottom velocity than that of granite between Bermuda and the West Indies, in order to explain its slightly lower velocity.

As for the differences in travel times of the Airy phases of Table II, it should be remembered that they are sensitive to differences in distance, since the velocity of propagation is comparatively low. Thus a difference in distance from the station to the epicenters of two shocks of 1° would cause a difference in travel times of the order of 1 min 40 sec. This is of the order of magnitude of the differences shown in Table II.

Bermuda is obviously ideally located for the recording of Airy phases of the Dominican Republic shocks, since the entire paths are oceanic. However a more favorable type of instrumentation would be highly desirable in order to make a thorough study of the Airy phase. In particular a vertical component seismometer, with a peak sensitivity in the vicinity of 11sec, a higher magnification and greater paper speed would be useful in obtaining more accurate data.

These Airy phases do not show up on the Milne-Shaw seismograms of the Dominion Observatory in Ottawa. Probably an appreciable amount of the energy of the phase is lost in the transfer from ocean to continent.

A fair sized shock occurred on the western flank of the Mid-Atlantic Ridge on May 6, 1944, at 22° .4 N 44.8° W (USCGS) H = 00:13.7. This appears to be the only one of the series of Mid-Atlantic Ridge quakes in recent years to have produced a recognizable Airy phase on the Bermuda seismograms. The amplidude of the group of waves interpreted as such is small; nevertheless this group is fairly well individualized and stands out well from the rest of the late coda. The period is 9 sec, and the arrival time 00:46:27, which means a travel time of 32 min 45 sec for Δ = 20° 17', i.e., a velocity of 3760 feet/sec, 0.75 of the velocity of sound in water. The double trace amplitudes in cm. read as follows:

| | NE | NW |
|---|---|---|
| P . . . . | 0.25 | 0.7 |
| S . . . . | 2.0 | 1.3 |
| A . . . . | 0.3 | 0.5 |

## 2. Huancayo

The Huancayo Magnetic Observatory is situated at 12°03'S, 75°20'W. It is equipped with 2 Wenner seismographs oriented NS and EW. The natural period is given as 10 sec and the magnification as 1550 by the USCGS seismological bulletin. It also has a Benioff vertical of period 1 sec and magnification unknown.

On Aug 5, 1944, both components of the Wenner recorded what appears to be the Airy phase of a shock which occured at 01:24:08 at 13 1/2°S 92 1/2°W (USCGS), south west of the Galapagos Is. The arrival time was 01:50:42, which gives a travel time of 26 min 34 sec. Since $\Delta = 17°$ the velocity comes out to be 3890 feet/sec or 0.8 of the velocity of sound in water. The period is 8 sec. Figure 17 shows this Airy phase. It is seen to be very conspicuous. The amplitudes in cm. are:

| | EW | NS |
|---|---|---|
| P . . . . | 1.1 | 0.3 |
| S . . . . | 1.4 | 2.0 |
| A . . . . | 1.0 | 0.7 |

It did not record on the Benioff vertical, the period of which is too short. This latter record proves conclusively that this is not a separate shock, but a phase of the quake recorded selectively only on the Wenner.

## Discussion

The velocities obtained for the Airy phase at Bermuda for the Oct 4, 1946, Dominican Republic and the May 6, 1944, Mid-Atlantic Ridge quakes and at Huancayo for the Aug 5, 1944, Galapagos shock are respectively 3480 ± 250 feet/sec, 3760 and 3890 feet/sec.

The second and third shocks were smaller than the first, so that in spite of the greater paths travelled in each, the estimated velocities cannot be considered as being any more accurate. The difference between the first two velocities, 3480 ± 250 and 3760 is therefore sufficiently small to be explainable by errors in the epicentral determinations.

As for the velocity obtained from the Galapagos shock, it should be remembered that Huancayo is situated a little over 150 miles inland, so that the velocity of 3890 feet/sec is probably a little high. It appears only reasonable to assume that once the Airy phase type motion reaches land it starts travelling with a velocity similar to the velocity of microseisms on land, roughly 3 times as fast as its velocity along an ocean path. Allowing for the path across land, the velocity of the Airy phase through the ocean for this shock is .7 of the velocity of sound in water. There are therefore no real discrepancies in our figures for the velocities.

As for the amplitudes of the various Airy phases, it appears that when the epicenter of the shock is in approximately the same azimuth as one of the components of the recording instrument, it is this component that comes out largest. This is the case for the Mid-Atlantic Ridge quake of May 6, 1944, (the NW component of the Airy phase is the largest), and for the Galapagos quake (the EW component is predominant). These two records appear therefore to suggest that in the horizontal plane the motion is mostly longitudinal. Because of their orientation the Bermuda records do not yield any information on that score (Table IIshows that the amplitudes are about the same on both components.)

We have already mentioned the fact that for the two large shocks of Nov 25, 1941, (37 1/2° N, 18 1/2° W, magn. 8 1/4) and April 11, 1946, (1°S, 14 1/2°W, magn. 7.2) the Bermuda Milne-Shaws failed to record any Airy phases. A bathymetric chart of the Atlantic Ridge. We believe that this provides an adequate explanation of the failure of the Airy phases to show up in the Bermuda instruments.

The Mid-Atlantic Ridge forms a continuous barrier across the Atlantic, 100 or more miles in width, rising well above the 2000 fathom line throughout its whole length, except for one or two narrow trenches. In sections it rises above the 1000 fathom line. It is to be expected that an Airy phase reaching this barrier would lose much of its energy. What would be left after crossing this barrier would not have enough amplitude to reach Bermuda.

As for shocks occuring on the western flank of the Ridge, all of those which occured in recent years would have been too small to produce an Airy phase with appreciable amplitude.

It also seems probable that the Airy phase does not propagate over distances of the order of several thousand miles, or at least, if it does, its amplitude is not large enough to distinguish it from the very late coda. Thus the torsion seismometers and linear strain seismometers of the west coast do not show any Airy phases for the Alaskan and Japanese shocks, and neither do the Honolulu Milne Shaws; the latter it is true, are troubled by a very high background.

## Mechanism of production of an Airy phase.

It can be shown that an impulse either in the liquid layer or in the bottom will produce waves that will propagate according to the laws of normal mode theory.

Thus any earthquake whose epicenter is at sea or close to the shore will produce, among other types of waves, an Airy phase after propagation for a considerable distance through the acoustic system consisting of ocean and bottom. However it will be only under certain given conditions that this phase will have sufficient energy to produce a noticeable effect on a seismometer situated at a certain distance.

It will be noted that all the shocks for which we have found Airy phases are shallow focus ones. The focus is therefore situated practically on the liquid-solid interface. The impulse on the liquid layer is strongest under these conditions. It is probable that the Airy phase produced by a shock whose focal depth is appreciably over 5-10 km. is too weak to be recorded at any distance.

All our data refer to records obtained by seismometers situated on land. It seems quite probable that with the aid of sensitive hydrophones in the ocean, at sufficient depth to remove the effect of surface gravity waves, it would be possible to record the Airy phases of a great many more shocks and at considerably greater distances.

## Practical applications

The study of the Airy phase of a sufficient number of sea quakes would provide interesting information concerning the nature of the ocean bottom, since the latter has an influence upon both the periods and the velocities of the Airy phase.

Conversely, once the velocities of the Airy phase are known for certain paths, the recording of these by means of suitable instruments could be useful for the more accurate determination of epicentral distances and epicenter locations.

## Acknowledgements

The authors wish to express their thanks to the offices of the U. S. Coast and Geodetic Survey, Section of Seismology, for making the Bermuda records available. Professors B. Gutenberg and C. F. Richter of the California Institute of Technology were extremely helpful during this investigation. We wish to thank Father Daniel Linehan, S.J., of the Weston Seismological Observatory, Dr. E. A. Hodgson of the Dominion Observatory of Canada, Professor Perry Byerly of the University of California, Father Joseph J. Lynch, S.J., and Dr. William A. Lynch of Fordham University for permission to study records of the respective observatories. We are indebted to Mrs. Marie Flanagan for computing and drafting the graphs presented in this paper.

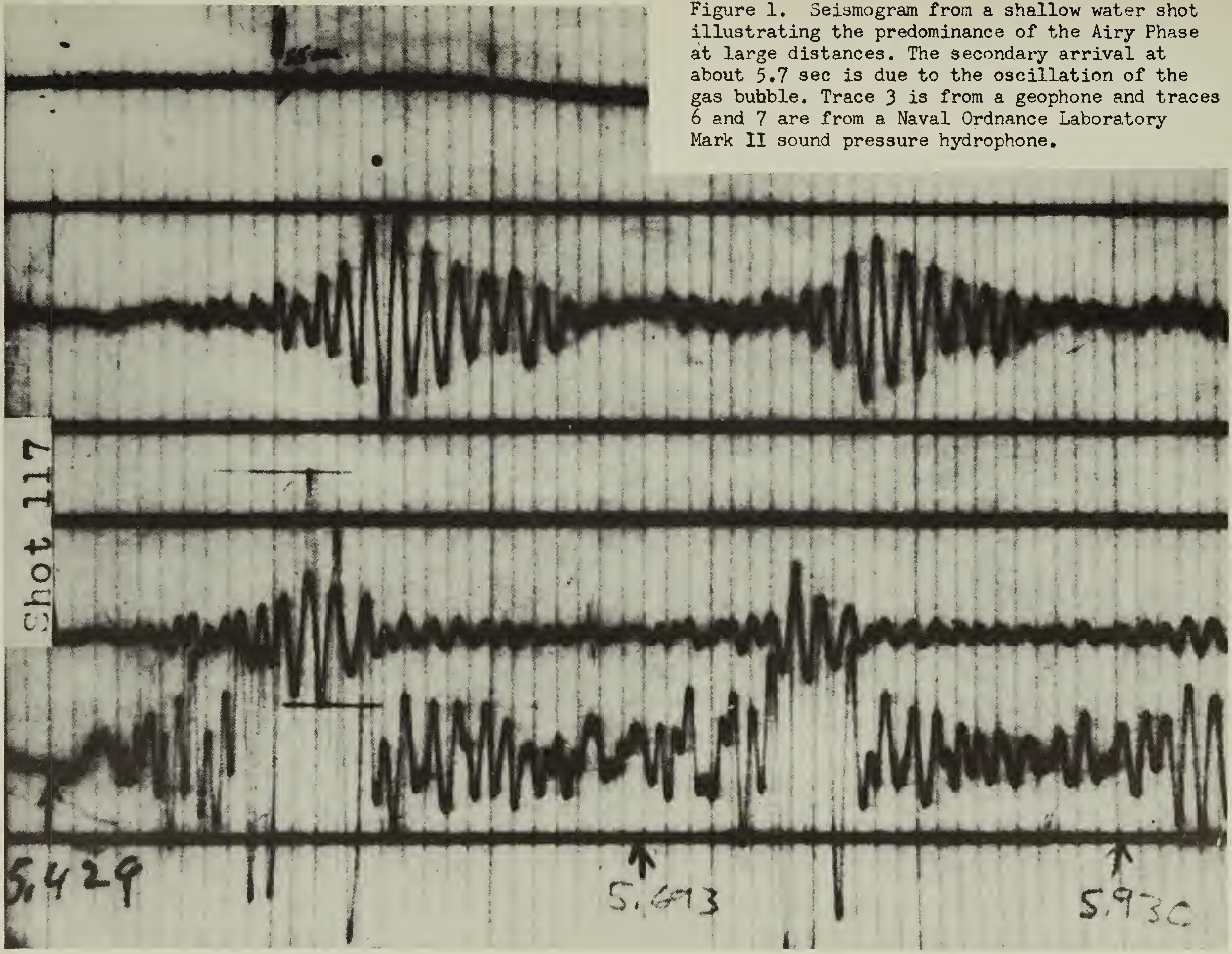


Figure 1. Seismogram from a shallow water shot illustrating the predominance of the Airy Phase at large distances. The secondary arrival at about 5.7 sec is due to the oscillation of the gas bubble. Trace 3 is from a geophone and traces 6 and 7 are from a Naval Ordnance Laboratory Mark II sound pressure hydrophone.

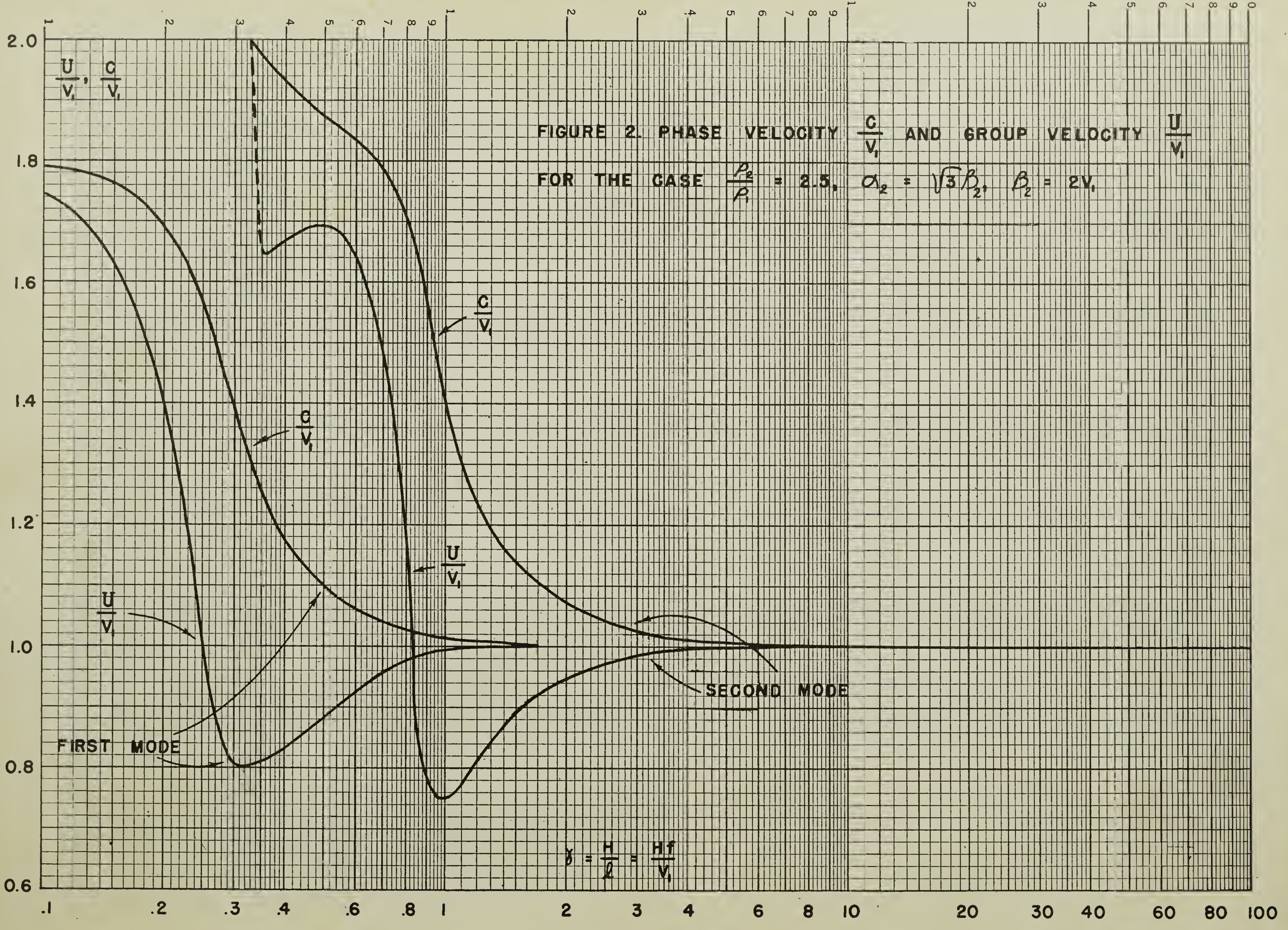


FIGURE 2. PHASE VELOCITY $\frac{C}{V_1}$ AND GROUP VELOCITY $\frac{U}{V_1}$

FOR THE CASE $\frac{\rho_2}{\rho_1} = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 2V_1$

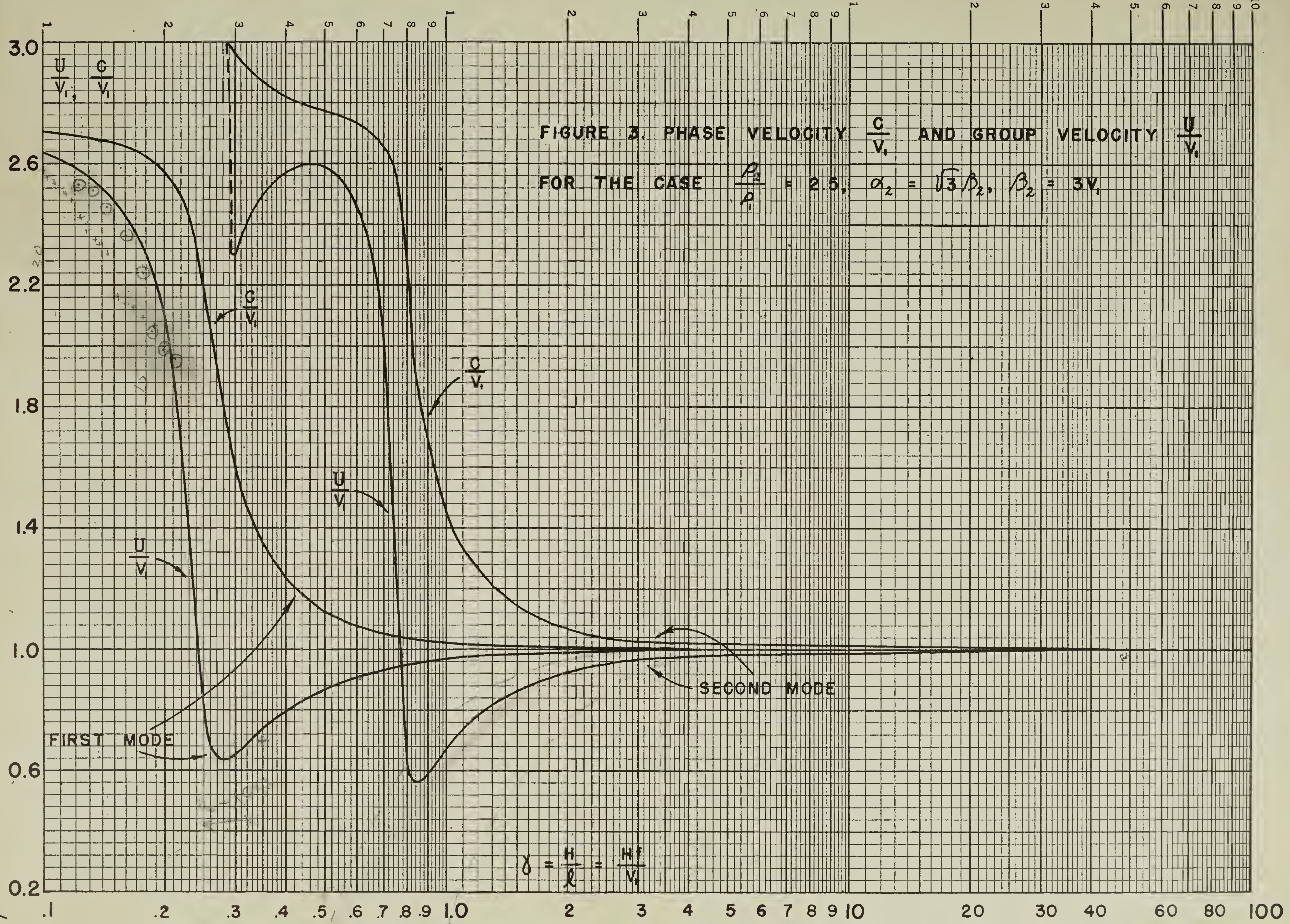


FIGURE 3. PHASE VELOCITY $\frac{C}{V_1}$ AND GROUP VELOCITY $\frac{U}{V_1}$

FOR THE CASE $\frac{\rho_2}{\rho_1} = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 3V_1$

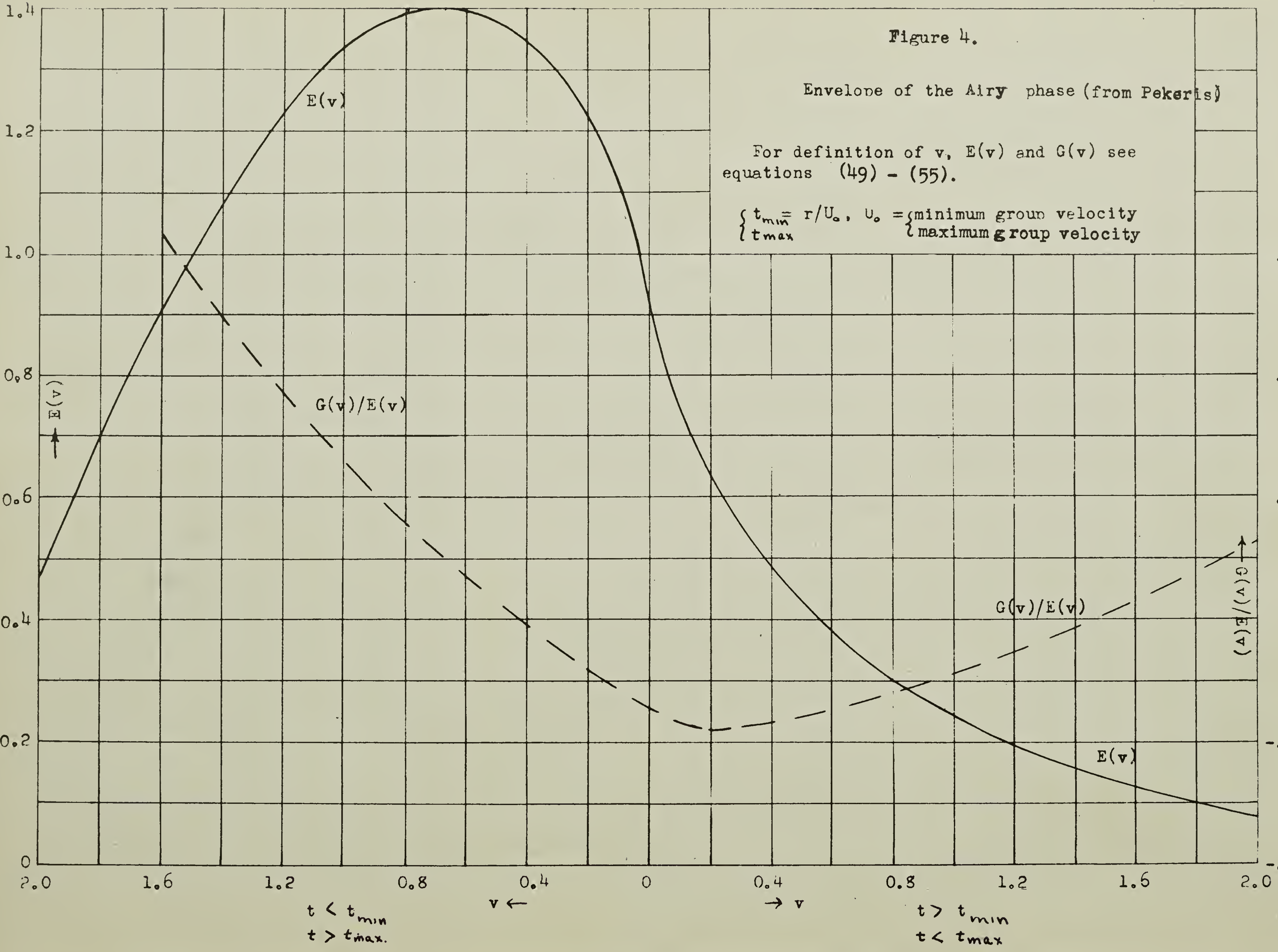


**Figure 4.**

Envelope of the Airy phase (from Pekeris)

For definition of v, E(v) and G(v) see equations (49) - (55).

$\left\{\begin{matrix} t_{min} \\ t_{max} \end{matrix}\right. = r/U_o$, $U_o = \left\{\begin{matrix}\text{minimum group velocity} \\ \text{maximum group velocity}\end{matrix}\right.$

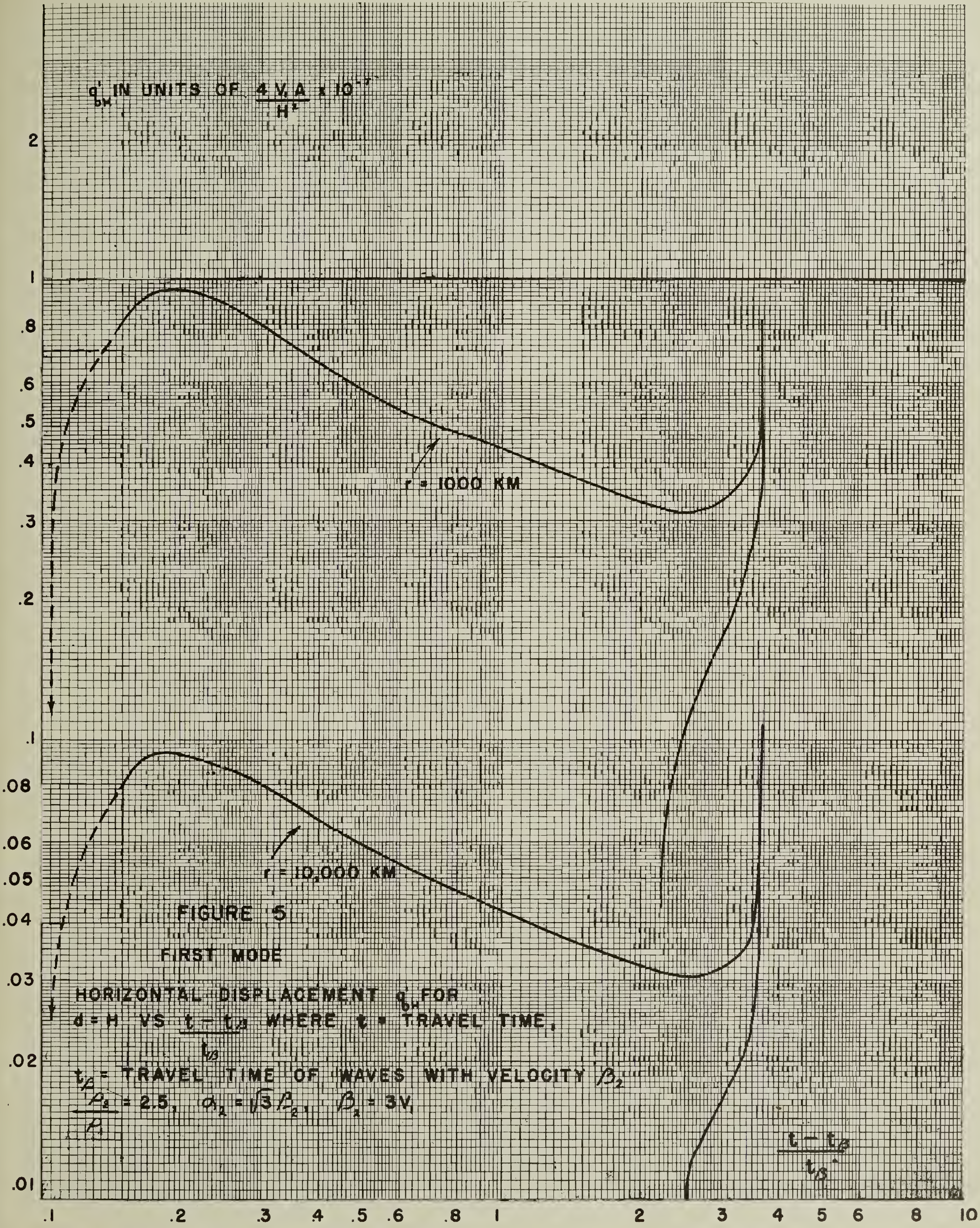


FIGURE 5

FIRST MODE

HORIZONTAL DISPLACEMENT $q_{bH}$ FOR $d = H$ VS $\frac{t - t_\beta}{t_\beta}$ WHERE $t$ = TRAVEL TIME,

$t_\beta$ = TRAVEL TIME OF WAVES WITH VELOCITY $\beta_2$

$\frac{\rho_2}{\rho_1} = 2.5$, $\alpha_2 = \sqrt{3}\,\beta_2$, $\beta_2 = 3V_1$

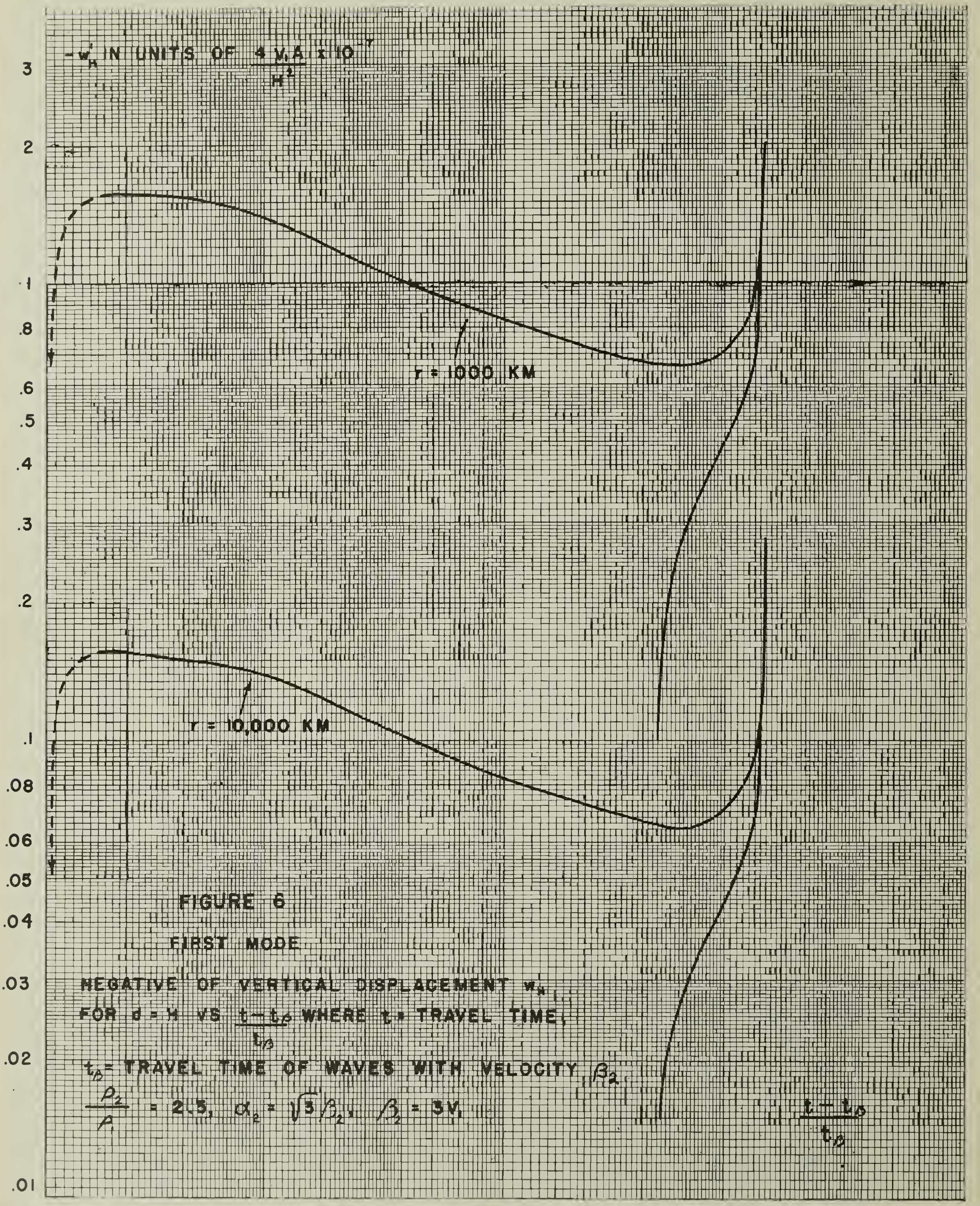


FIGURE 6

FIRST MODE

NEGATIVE OF VERTICAL DISPLACEMENT $w_H$ FOR d = H VS $\frac{t-t_\beta}{t_\beta}$ WHERE t = TRAVEL TIME.

$t_\beta$ = TRAVEL TIME OF WAVES WITH VELOCITY $\beta_2$

$\frac{\rho_2}{\rho_1} = 2.5, \quad \alpha_2 = \sqrt{3}\,\beta_2, \quad \beta_2 = 3V_1$

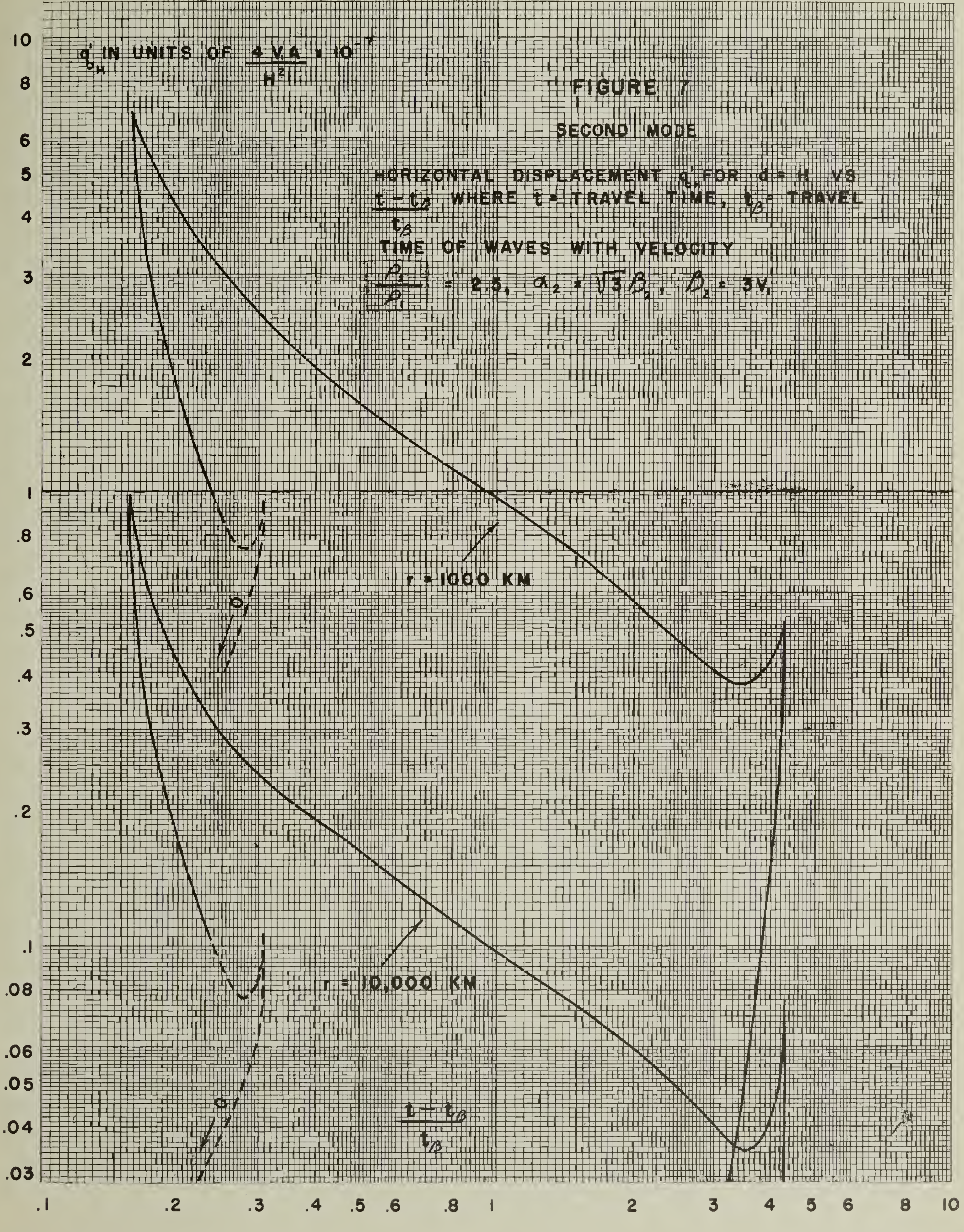
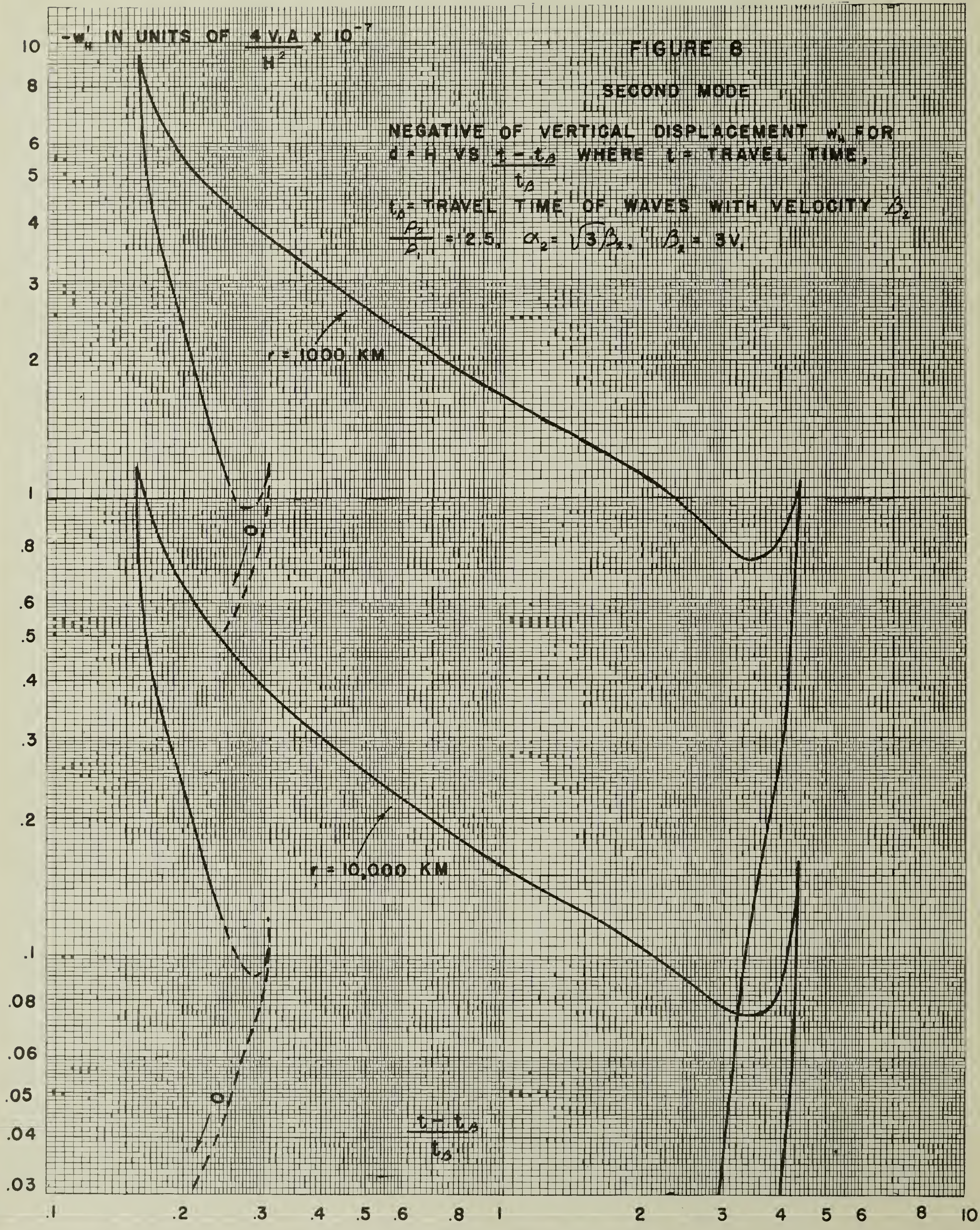


FIGURE 7

SECOND MODE

HORIZONTAL DISPLACEMENT $q'_{o_H}$ FOR d = H VS $\frac{t-t_\beta}{t_\beta}$ WHERE t = TRAVEL TIME, $t_\beta$ = TRAVEL TIME OF WAVES WITH VELOCITY

$\frac{\rho_2}{\rho_1} = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 3V_1$



## FIGURE 8

SECOND MODE

NEGATIVE OF VERTICAL DISPLACEMENT $w_H'$ FOR $d = H$ VS $\frac{t - t_\beta}{t_\beta}$ WHERE $t$ = TRAVEL TIME,

$t_\beta$ = TRAVEL TIME OF WAVES WITH VELOCITY $\beta_2$

$\frac{\rho_2}{\rho_1} = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 3V_1$

MADE IN U.S.A.

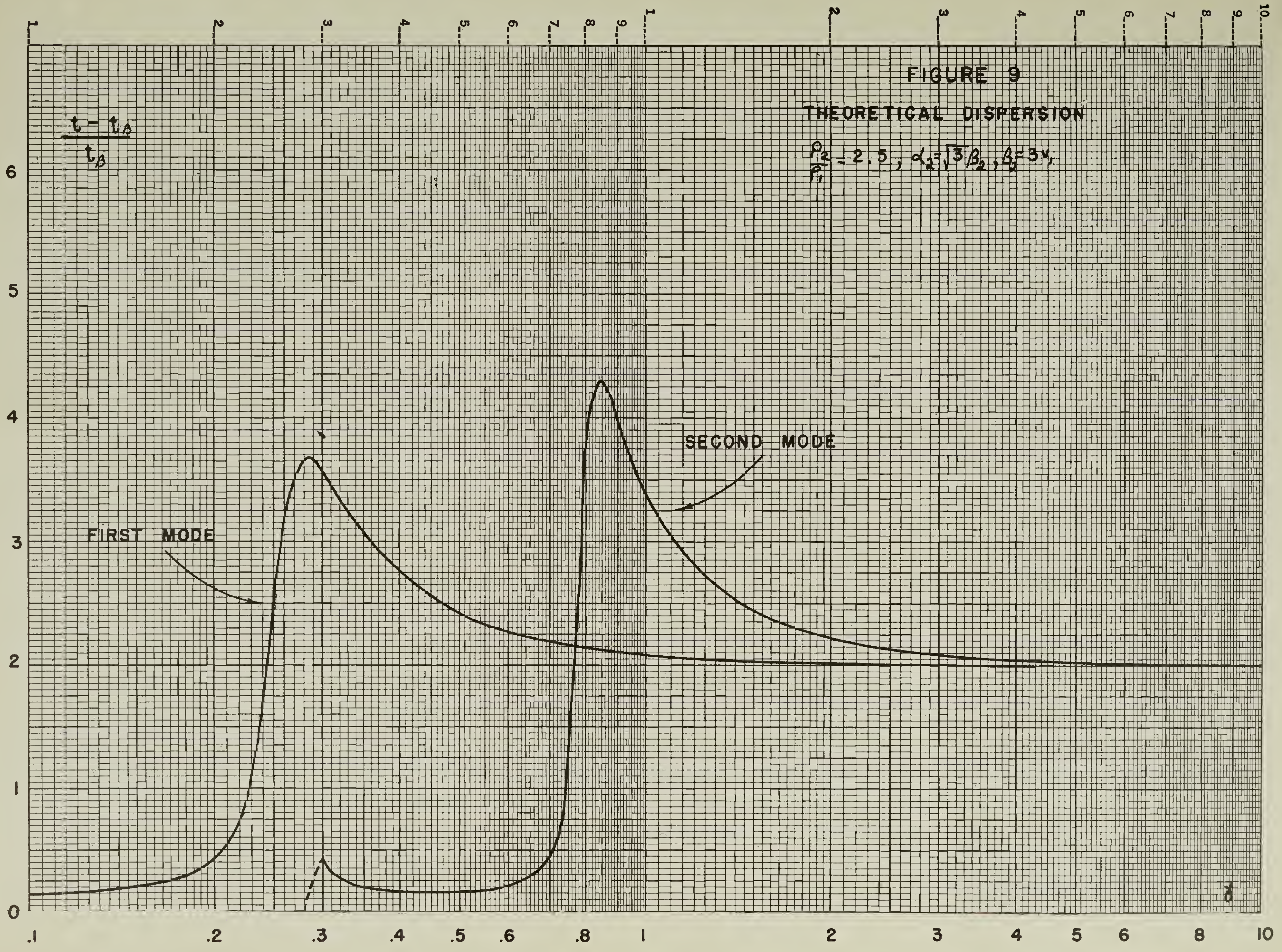


FIGURE 9

THEORETICAL DISPERSION

$\frac{\rho_2}{\rho_1} = 2.5, \ \alpha_2 = \sqrt{3}\beta_2, \ \beta_2 = 3\nu,$

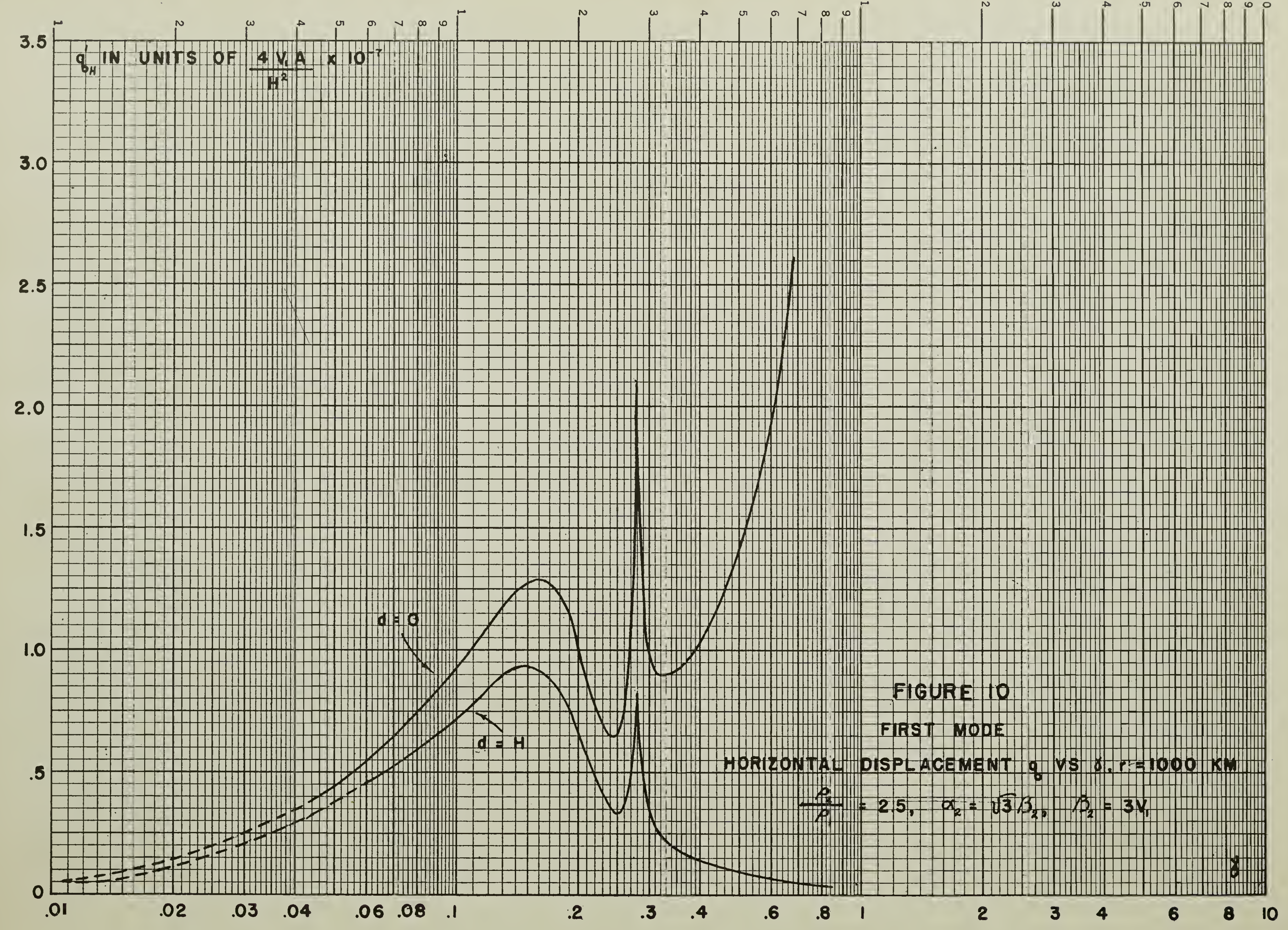


FIGURE 10

FIRST MODE

HORIZONTAL DISPLACEMENT $q_0$ VS $\delta$, r = 1000 KM

$\frac{\rho_2}{\rho_1} = 2.5, \quad \alpha_2 = \sqrt{3}\beta_2, \quad \beta_2 = 3V_1$

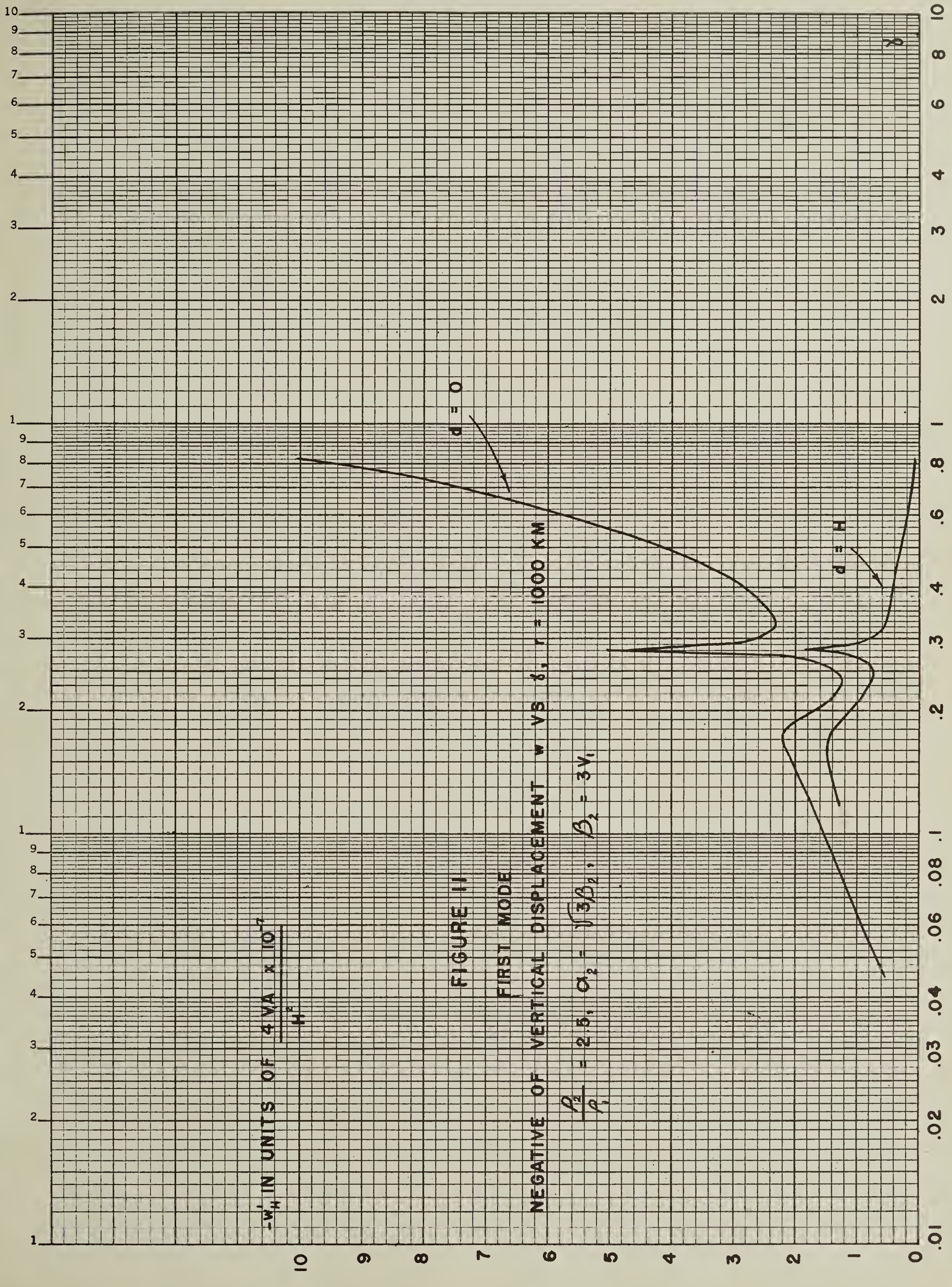


FIGURE 11

FIRST MODE

NEGATIVE OF VERTICAL DISPLACEMENT w VS δ, r = 1000 KM

$\frac{\rho_2}{\rho_1} = 2.5,\ \alpha_2 = \sqrt{3}\beta_2,\ \beta_2 = 3V_1$

$-w'_H$ IN UNITS OF $\frac{4VA}{H^2} \times 10^{-7}$

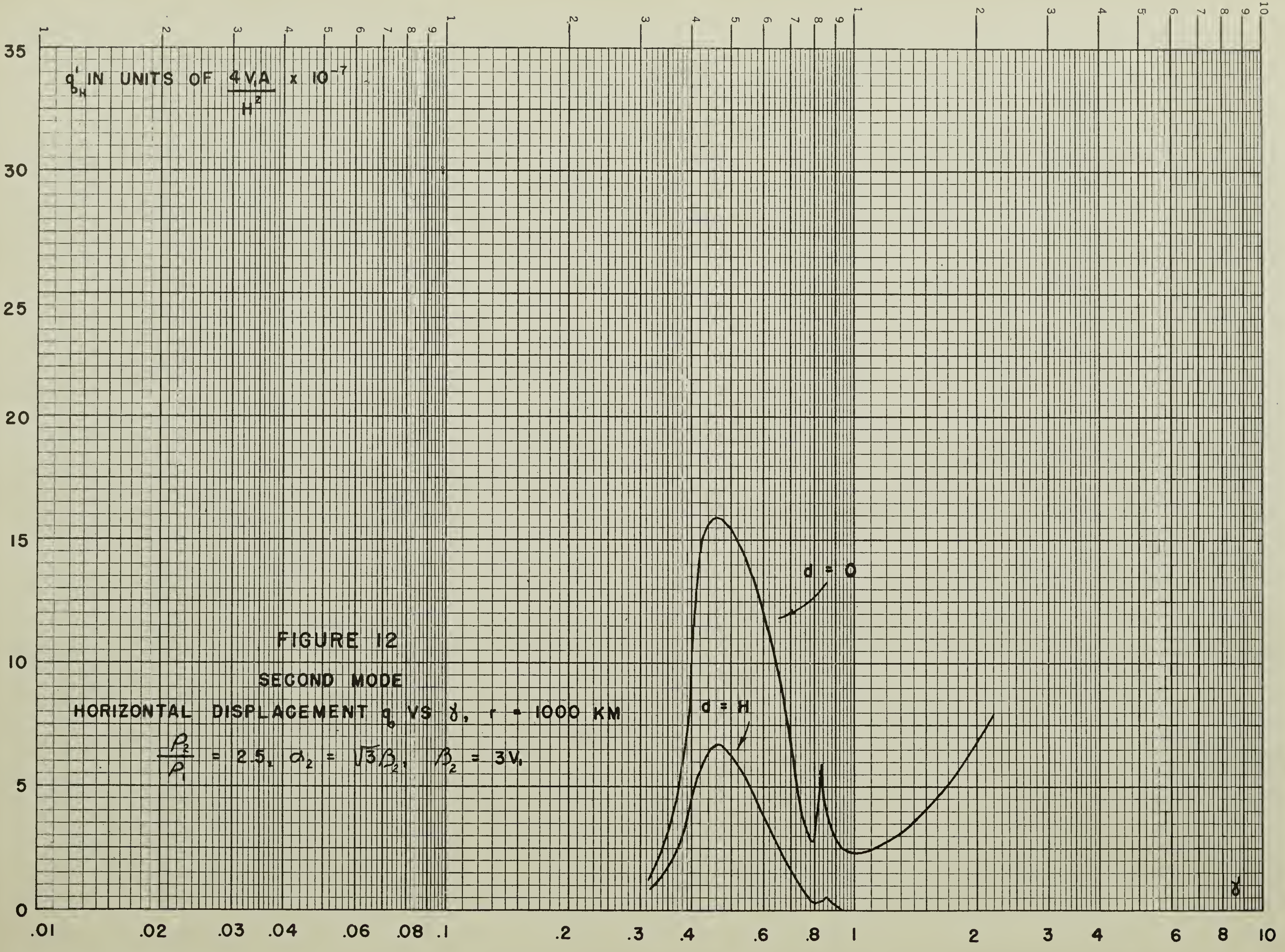


FIGURE 12

SECOND MODE

HORIZONTAL DISPLACEMENT $q_b$ VS $\gamma$, $r = 1000$ KM

$\frac{\rho_2}{\rho_1} = 2.5$, $\alpha_2 = \sqrt{3}\beta_2$, $\beta_2 = 3V_1$

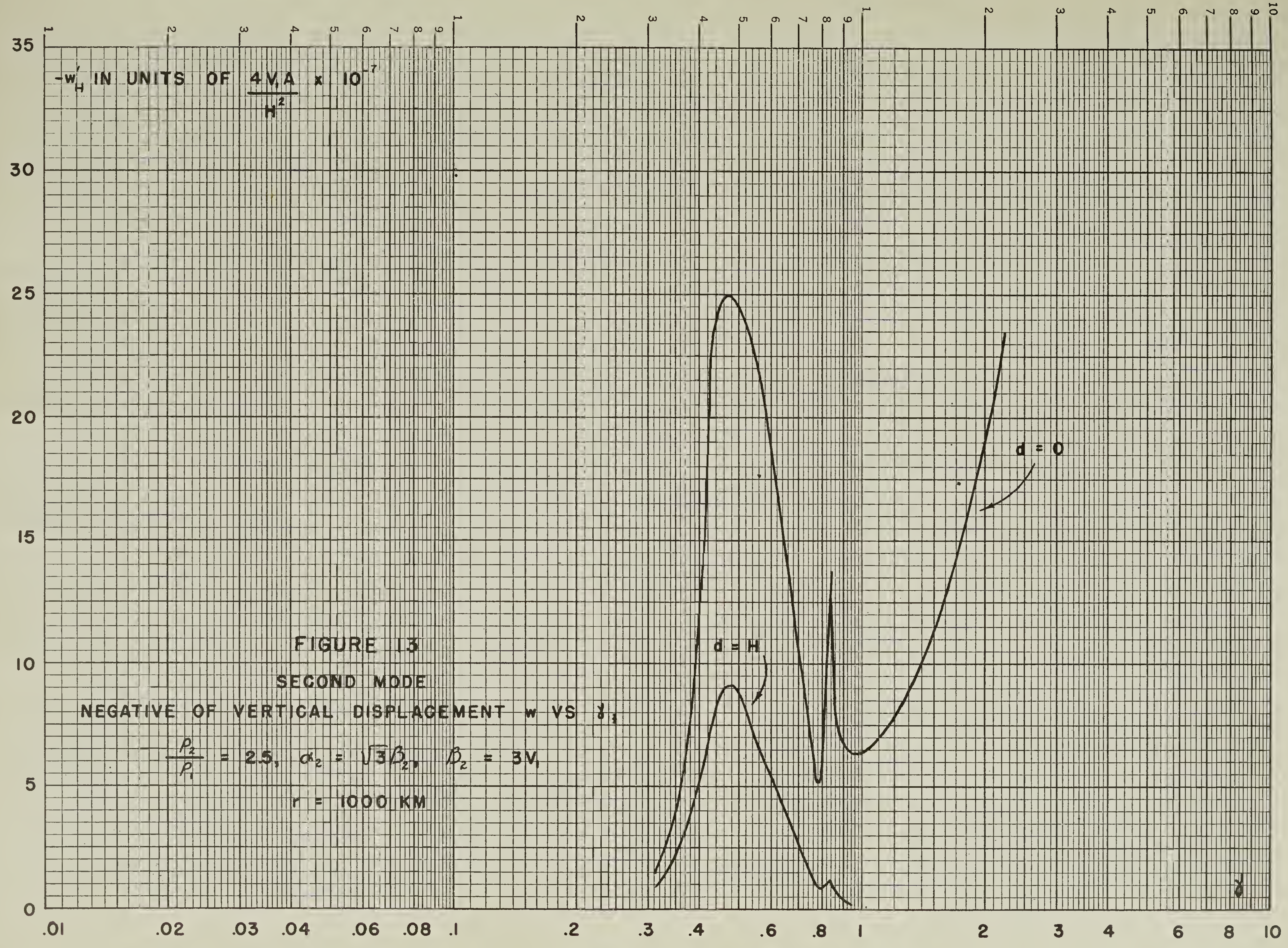


FIGURE 13

SECOND MODE

NEGATIVE OF VERTICAL DISPLACEMENT w VS $\gamma_1$

$\frac{\rho_2}{\rho_1} = 2.5, \quad \alpha_2 = \sqrt{3}\beta_2, \quad \beta_2 = 3V_1$

r = 1000 KM

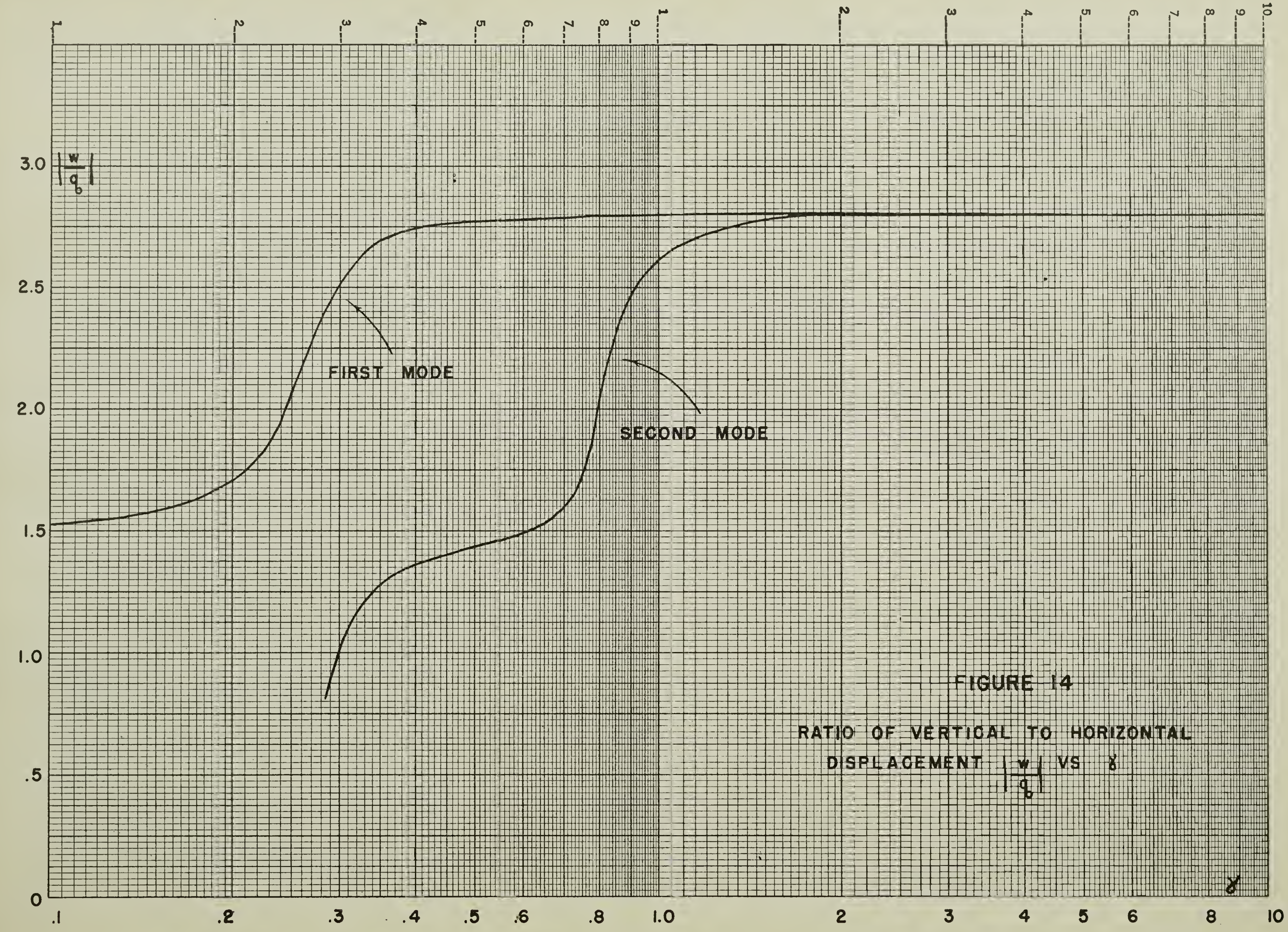
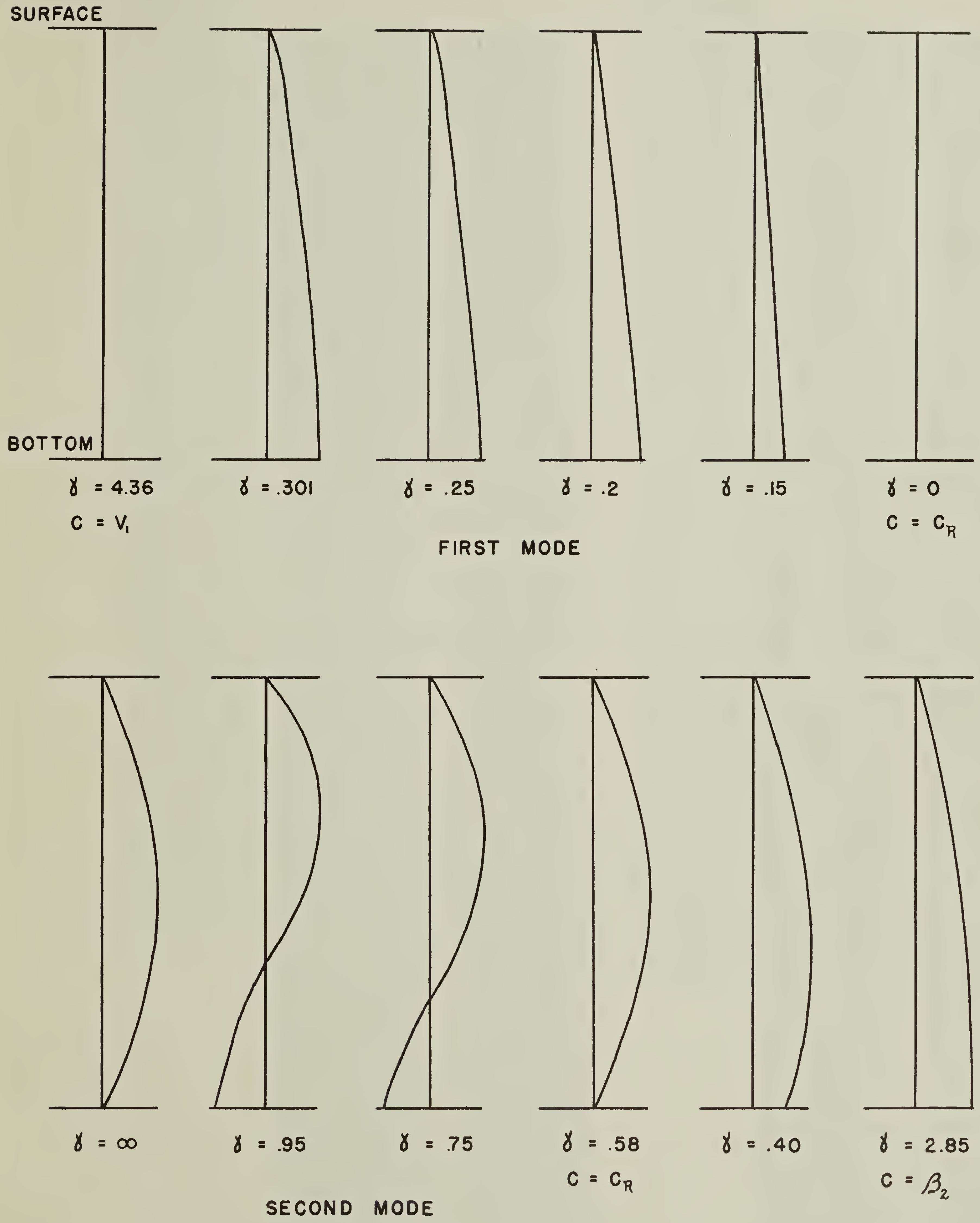


FIGURE 14

RATIO OF VERTICAL TO HORIZONTAL DISPLACEMENT $\left|\frac{w}{q_0}\right|$ VS $\gamma$



VERTICAL PRESSURE DISTRIBUTION AS A FUNCTION OF $\gamma$

FIGURE 15

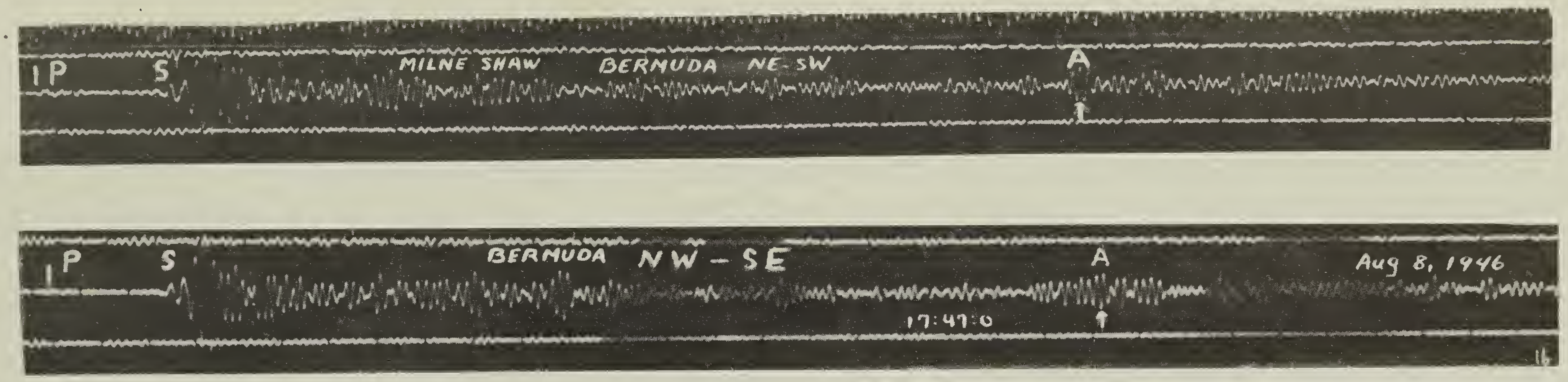


Figure 16a

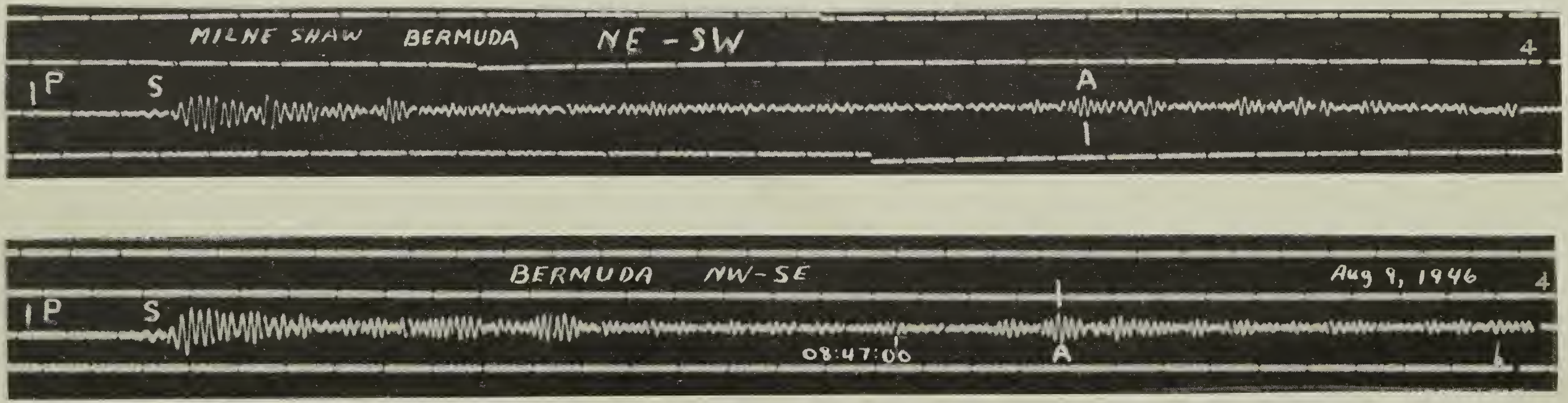


Figure 16b

Figure 16. Airy Phases recorded at Bermuda for the earthquakes a)1946 Aug $8^d$ $17^h$ and b) 1946 Aug $9^d$ $8^h$.

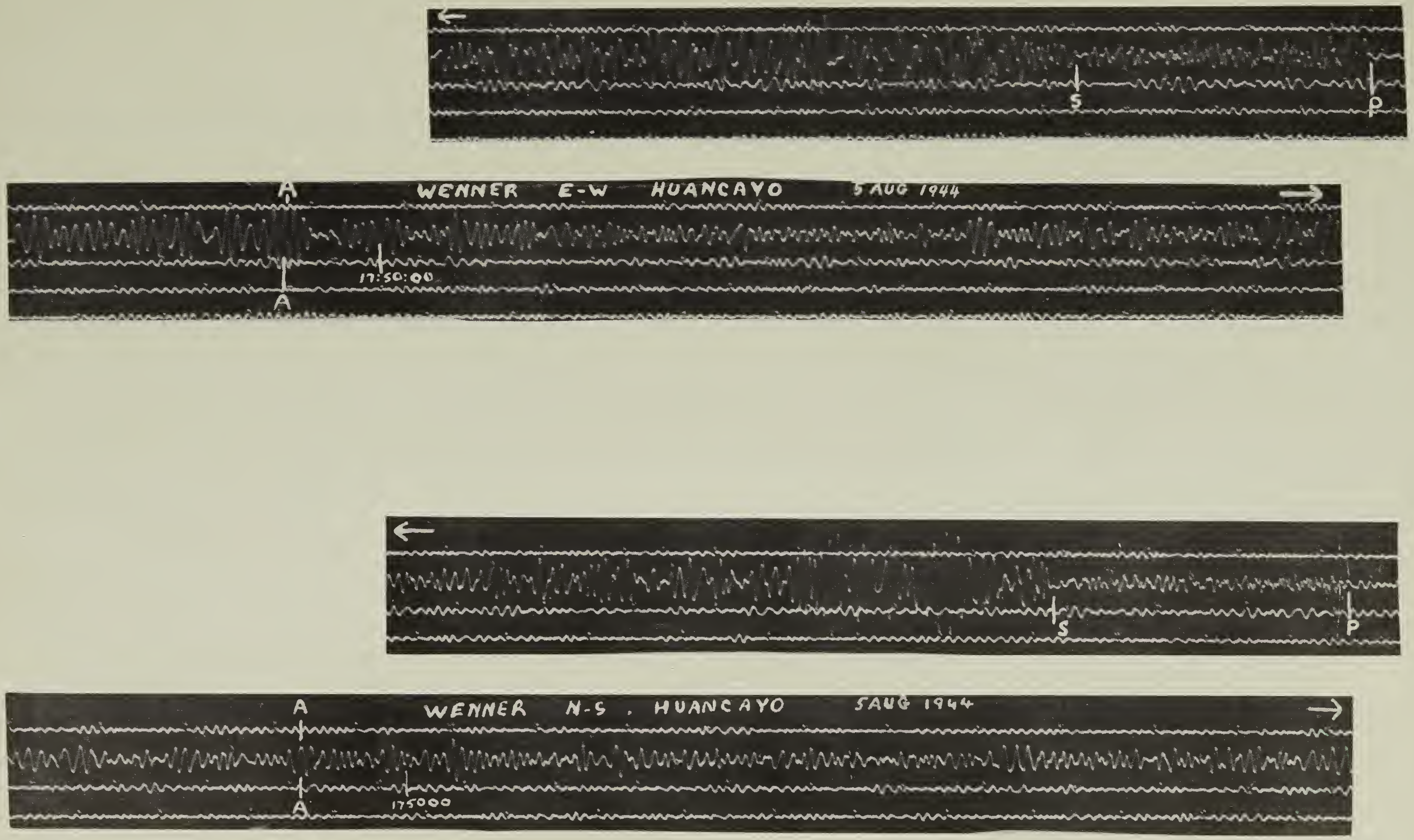


Figure 17. Airy Phase recorded at Huancayo for the earthquake of 1944 Aug $5^d$ $17^h$.

COLUMBIA LIBRARIES OFFSITE
CU90645880